Development of a Method for the Characterisation of the Compliance and Stiffness Matrices of Uncoupled Laminates

By

Carine Gachon

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A mes parents
(to my parents)
Evelyne et Gilbert

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ABSTRACT

The mechanical behaviour of composite materials is defined by three matrices of stiffness/compliance : in-plane, flexure and coupling matrix between in-plane and flexure. The present work gives a method of test to characterise the in-plane compliance matrix of uncoupled laminates. The matrix depends on the direction, hence the polar coordinates method, introduced by Verchery and Vong [2], is used to simplify the analysis. The strain-stress relation in polar coordinates, and the relationship between the compliance and the stiffness polar components are introduced. Using these relations, a system of six equations, having the elasticity coefficients (in polar) as unknown, is obtained. A classic tensile test method is used. A constant stress is applied to three samples cut in three different directions and the strains are measured (stress and strain are the inputs of the system). Four numerical examples are given showing the accuracy of the calculation. The problem is mainly the accuracy of the measurement. The interest in using four strain gages instead of three is studied as well as the consequence of an experimentation error on the strain state calculated. Finally, a program solving the system is presented as a tool for the experimental analysis.

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A-THEORETICAL BACKGROUND

I INTRODUCTION TO COMPOSITE MATERIALS

1-Definition

A composite material is created by combining two or more materials for the purpose of predictably enhancing certain properties.

Nature was the first to show us the interesting properties of composite materials. Thus, for example, many tissues in the body, which have high strength combined with enormous flexibility, are made up of stiff fibres such as collagen embedded in a lower stiffness matrix. Similarly, a microscopic examination of wood and bamboo reveals a pronounced fibrillar structure.

In the field of advanced composites materials, the combination consists of a relatively stiff, high strength reinforcing material, embedded within a relatively compliant, low strength matrix material.

2-Classification

There are three major classifications of composites : fibrous, laminar, and particulate. They are described below and shown in Fig.1 :

1) Fibrous composites are materials containing reinforcing fibres bonded to a matrix filler material. Fibres are very small in diameter and provide little or no strength or stiffness except in tension. Generally, a smaller diameter means fewer dislocation and instabilities within the fibre materials and, consequently, higher tensile strength. Many different materials are presently used as fibres, including glass, carbon, boron, graphite and tungsten. Glass is by far the most widely used reinforcement fibre and is the lowest cost. The strength is mainly related to the arrangement of fibres.

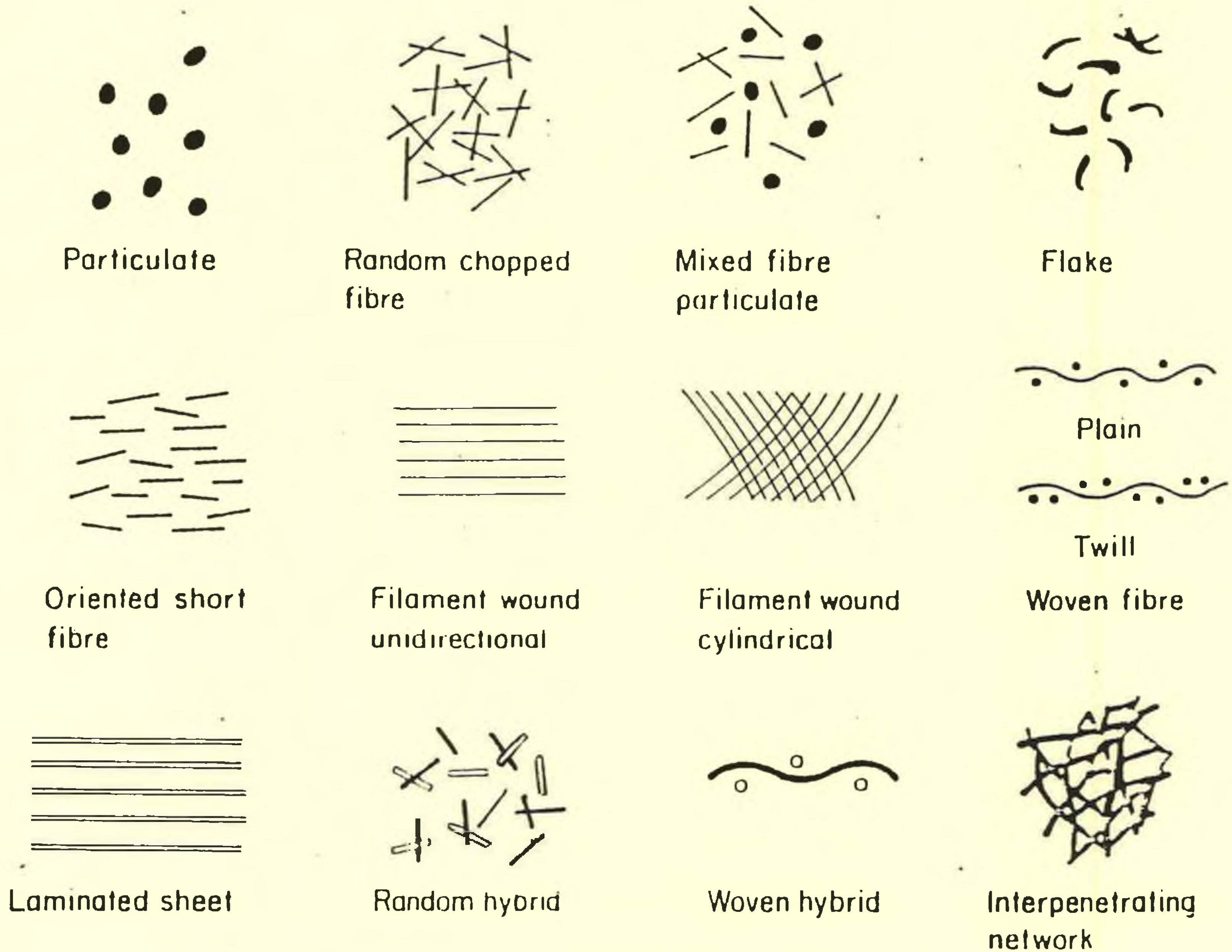


Fig. 1 Examples of different composite geometrical arrangements.

2) Laminar composites are composed of layers of materials bonded together. This category includes both sandwich and honeycomb composites as well as several types of wooden layered composites. A major area of study includes composite laminates which are composed of several elementary parallel orthotropic plies, or laminae, strongly bonded together. The plies have a typical constant thickness of a fraction of millimetre and are considered as homogeneous on the scale of any laminate element
3) Particulate composite consists of particles dispersed in a matrix. The types of particles can be either skeletal or flake and a wide variety of sizes, shapes and materials are available.

Example of matrices and fibres :

Matrices	Fibres
Polyesters Epoxides Polyimides Rubber/Polyurethane modified Epoxides Xyloks Bismaleimides (BMI) Polystyryl Pyridine (PSP)	Aromatic polyamid (Aramid) Boron Carbon Glass

3-Comparison Between Composite And Conventional Materials

There has been a quick growth in the use of fibre reinforced materials in engineering applications in the last few years and there is every indication that this will continue. The rapid growth has been achieved mainly by the replacement of traditional materials, mainly metal and wood. On the basis of strength and stiffness alone, fibre reinforced composite materials do not have a clear advantage particularly when is noted that their elongation to fracture is much lower than metals with comparable strength. The advantages of composite materials appear when the modulus per unit weight (specific modulus) and strength per unit weight (specific strength) are considered. That means that due to higher specific modulus and specific strength, the weight of components made of composite materials can be reduced.

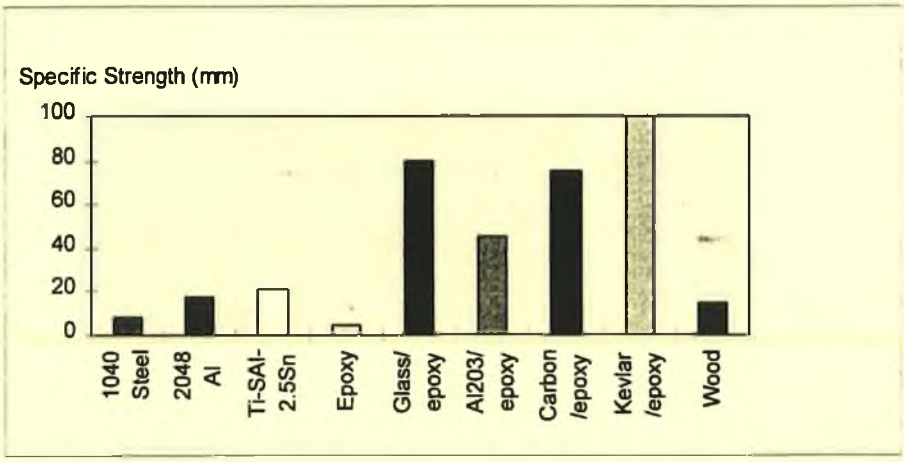


Fig. 2 - Comparison of specific strengths between some usual engineering material

The very large difference in different directions may be a serious limitation in some applications since the material will be highly anisotropic. However, it is also a source of one of the outstanding advantages of composite materials since it allows the possibility of introducing stiffness and strength into a product where it is a particular requirement.

II CLASSICAL LAMINATED PLATE THEORY [1]

1-Introduction

A typical laminated plate small element is depicted in Figure 2 where the 3 axis is perpendicular to the 1-2 plate mid-plane.

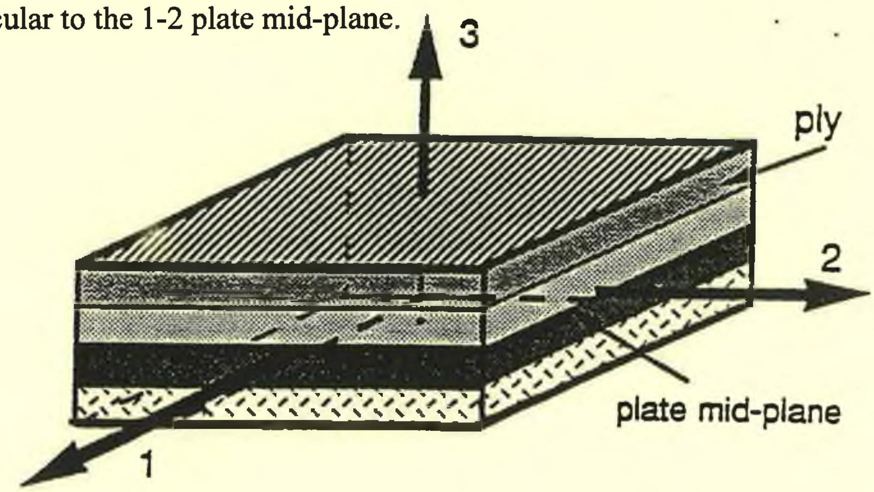


Fig. 3 - Laminate coordinate system

The plies mechanical properties can be characterised by effective engineering constants directly derived from the constituents ones . Let x(p), y(p), z(p) be the on-axis ply p coordinates and 1, 2, 3 the laminate reference basis such as 1, 2 is the plate mid-plane . The z(p) and 3 axes are both normal to the mid-plane and the ply p orientation is defined by the angle α(p) between x(p) and 1 axes.

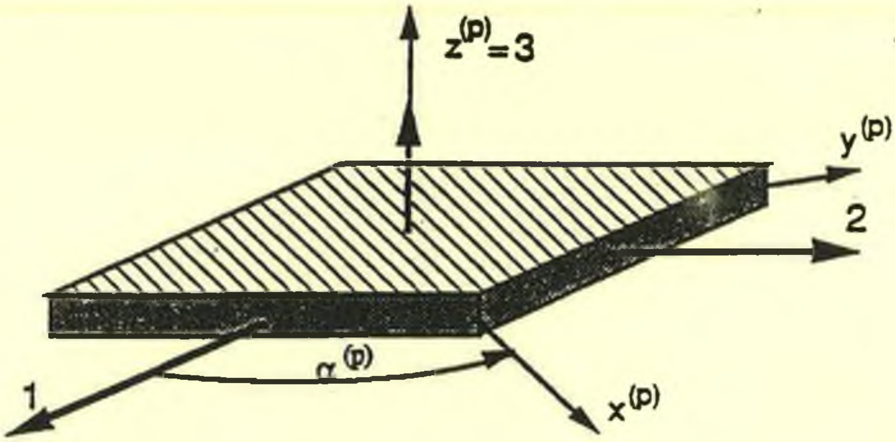


Fig. 4 - Symmetry axes of ply p

2-Basic Assumptions of the Classical Laminated Plate Theory.

1- Ply behaviour is linear orthotropic and hygro-thermoeslastic.

2- Ply bonding is perfect.

3- Strains and displacements remain small

4- Body forces are neglected (i.e. traction vectors are prominent).

5- Transverse plate loading is normal to the plate mid-plane.

6- Each ply is in a state of plane stress in the 1,2 plane.

7- Love- Kirchhoff assumptions are applied (thin plate)

- The normal to the initial mid-plane is transformed into the normal to the deformed mid-plane.
- The plate thickening is neglected.

Assumption 2 implies the continuity of the displacement and stress vectors at the various ply interfaces .

Assumption 6 (concerning the ply) means that the effect of the plane stresses is prominent; therefore the constitutive relations of every ply will be written in plane stress. Such an assumption is quite justified by the fact that the transverse shear stresses σ_{i3} (for i = 1, 2) are equal to zero on the outer laminate surfaces (assumption 5) and that the laminate thickness is small. Although the material internal equilibrium obviously requires a three dimensional state of stress, this very simple approach does not take into account the occurrence of important σ_{i3} (for i = 1, 2, 3).

Assumption 7 (concerning the laminate) simply asserts that the transverse effect can be first neglected compared to the in-plane effects. Although neglecting transverse shear deformation effects may sometimes lead to significant errors (in particular dealing with sandwich structures), in many cases, neglecting them results in easier solutions which are quite useful in preliminary design.

3-Elastic Behaviour of a Ply in Plane Stress

Assuming that the 1, 2 plane is the stress plane, then :

$\sigma_{33} = \sigma_{23} = \sigma_{13} = 0$ (i.e. $\sigma_3 = \sigma_4 = \sigma_5 = 0$ in contracted notations)

Then, the in-plane stress-strain relationship of ply p in the ply on-axis co-ordinates $x^{(p)}$, $y^{(p)}$ is as follows :

$$\{\varepsilon^{(p)}\} = [s^{(p)}]\{\sigma^{(p)}\} + \{\varepsilon_{ht}^{(p)}\} \qquad (1)$$

Where :

$$\{\varepsilon^{(p)}\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{Bmatrix}, \qquad \{\sigma^{(p)}\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix}, \qquad [S^{(p)}] = \begin{bmatrix} S_{xx} & S_{xy} & 0 \\ S_{xy} & S_{yy} & 0 \\ 0 & 0 & S_{ss} \end{bmatrix}$$

$\{\varepsilon_{ht}^{(p)}\} = \{\alpha^{(p)}\}\Delta T + \{\beta^{(p)}\}\Delta c$ are the hygrothermal strains such as :

$$\{\alpha^{(p)}\} = \begin{Bmatrix} \alpha_x \\ \alpha_y \\ 0 \end{Bmatrix}, \qquad \{\beta^{(p)}\} = \begin{Bmatrix} \beta_x \\ \beta_y \\ 0 \end{Bmatrix}$$

ΔT and Δc stand for the homogeneous variations of temperature and moisture concentration through ply p; α_x and β_x are respectively the temperature and moisture expansion of the material in $x^{(p)}$-axis.

The inversion of equation (1) provides the stress-strain relationship in plane stress in terms of reduced stiffness :

$$\{\sigma^{(p)}\} = [Q^{(p)}](\{\varepsilon^{(p)}\} - \{\varepsilon_{ht}^{(p)}\}) \qquad (2)$$

Where $[Q^{(p)}]$ is the reduced stiffness matrix for plane stress of ply p in the ply on-axis coordinates $x^{(p)}$, $y^{(p)}$; it is obviously obtained by simple inversion of the previous compliance matrix $[S^{(p)}]$ and we have :

$$[Q^{(p)}] = \begin{bmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{xy} & Q_{yy} & 0 \\ 0 & 0 & Q_{ss} \end{bmatrix}$$

It can be derived from the three dimensional stiffness matrix $[C^{(p)}]$ as well and the relationship below defines the reduced stiffness versus the three dimensional stiffness:

$$Q_{ij} = C_{ij} - C_{i3}\frac{C_{j3}}{C_{33}} \qquad (3)$$

4-Laminated Plate Theory

The direct solution of the well known differential system defining the strains as functions of the displacement vector immediately leads to the following expression of the in-plane laminate strains in the 1,2 basis :

$$\{\varepsilon\} = \{\varepsilon^0\} + x_3\{k\} \qquad (4)$$

where :

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix}, \quad \{\varepsilon^0\} = \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_3^0 \end{Bmatrix}, \quad \{k\} = \begin{Bmatrix} k_1 \\ k_2 \\ k_3 \end{Bmatrix} = -\begin{Bmatrix} w_{,11} \\ w_{,22} \\ 2w_{,12} \end{Bmatrix}$$

$\{\varepsilon^0\}$, $\{w\}$ and $\{k\}$ are respectively the in-plane strains, the deflection and the curvatures of the mid-plane ; these three functions only depend on x_1 and x_2. Therefore the laminate in-plane strains$\{\varepsilon\}$ are linear functions of x_3 (equation (4)). Finally it follows that the in-plane ply stresses $\{\sigma^{(p)}\}$(in the symmetry ply axes) or $\{\sigma^{(p)}\}$(in the laminate axes) are linear functions of x too.

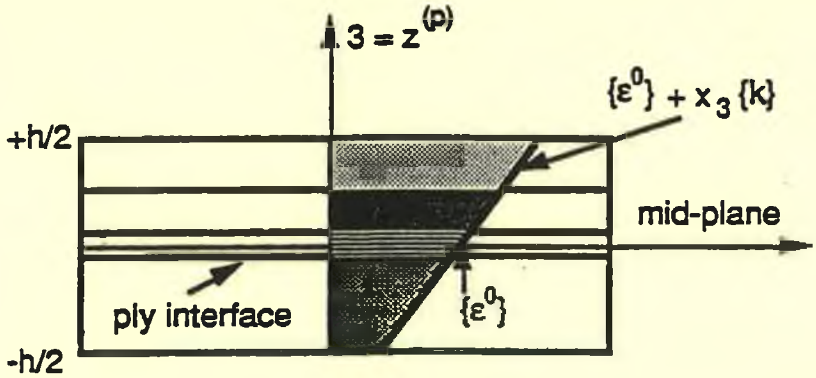


Fig. 5 - In-plane laminate strains

It is worth noting that the in-plane strains and displacements are necessarily continuous functions of the x_1, x_2, x_3 variables as soon as the ply bonding is perfect; but in general there is no justification at all for ε_3, ε_4, ε_5 (i.e. ε_{33}, ε_{23}, ε_{13} strain tensor components) to be continuous at the ply interfaces.

In conclusion, 6 strain components characterise the deformation state of a thin plate, namely 3 in-plane mid-plane strains $\{\varepsilon^0\}$ and 3 mid-plane curvatures $\{k\}$ which only depend on x_1, x_2 variables.

5-Resultant Stresses and Moments

Dealing with plates, the control element of volume is a rectangular parallelepiped of dimension dx_1, dx_2 and h, the total constant thickness of the laminate. This control element determines the observation scale associated with the plate modelling. In particular it implies an homogenisation through the thickness of the laminate which is necessary since our purpose is now to establish stress-strain relationships for plates (and not for materials).

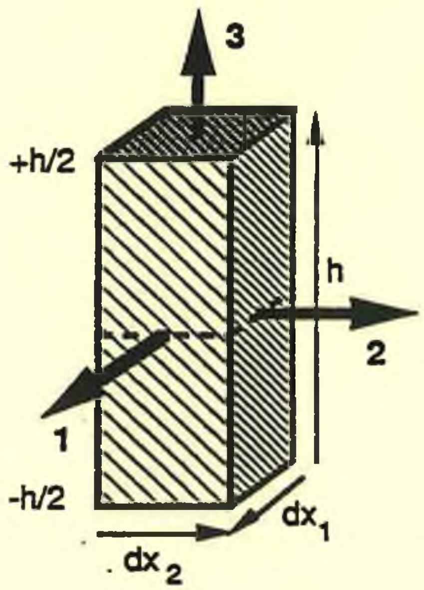


Fig. 6 - Plate control element

It is obvious that the final resulting mechanical description will be less refined than a model based on a three dimensional element dx_1, dx_2, dx_3. However, in the framework of thin plate assumption, it will lead to a first satisfactory sizing of the structure.

The stress distribution acting on any elementary lateral face of the plate is simply modelled by the resultant stresses and the stress moments obtained by integration of the ply stress components over the thickness h, regardless of the number and orientation of the plies, hence :

$$\{N_m\} = \int_{-h/2}^{+h/2} \{\sigma\} dx_3 , \quad \{M_m\} = \int_{-h/2}^{+h/2} \{\sigma\} x_3 dx_3 \qquad (5)$$

in the laminate basis,

$\{\sigma\}=\{\sigma^{(p)}\}$ for x_3 belonging to ply p.

The matrices $\{N_m\}$ and $\{M_m\}$ are respectively the purely mechanical stress resultants (homogeneous to a force per length unit) and the stress moments.

The ply stress distribution (a) over the plate thickness is replaced by the linear laminate stress distribution (b) to calculate the plate strains $\{\varepsilon^0\}$ and $\{k\}$.

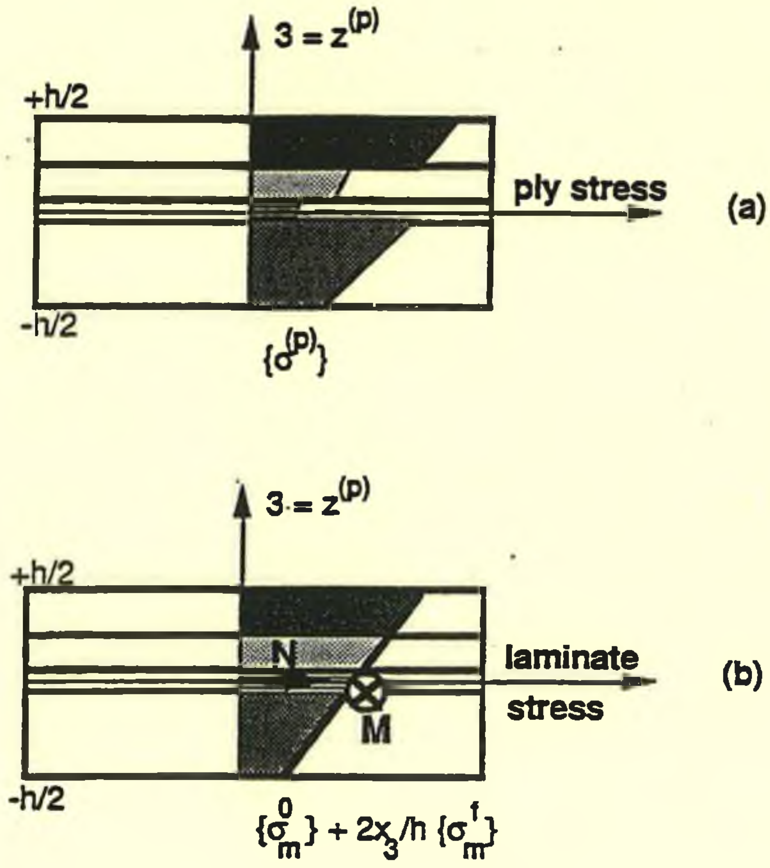


Fig. 7 - Ply stress (a) and its equivalent laminate stress (b)

Laminate analysis :

The laminate stress-strain relations are directly derived by substituting {σ}=[Q]{ε} in equation (2) into equation (5).

$$\{N_{m}\} = [A]\{\varepsilon^0\} + [B]\{k\} - \{N_{ht}\} \qquad (6)$$
$$\{M_{m}\} = [B]\{\varepsilon^0\} + [D]\{k\} - \{M_{ht}\}$$

in the laminate coordinate system.

The 3x3 symmetrical matrices [A], [D] and [B] are respectively the in-plane stiffness matrix, flexural stiffness matrix and coupling matrix between in-plane and flexure; the vectors $\{N_{ht}\}$ amd $\{M_{ht}\}$ are the hygrothermal stress resultant and moment. These different quantities are linear functions of the ply characteristics $[Q^{(p)}]$, $\{\alpha^{(p)}\}$and $\{\beta^{(p)}\}$.

We now define the normalised stiffness [A*], [D*] and [B*] in Pa as follows :

$$[A^*] = \frac{1}{h}[A]\ ,\ [D^*] = \frac{12}{h^3}[D],\quad [B^*] = \frac{2}{h^2}[B]$$

hence the effective stress-strain relations become :

$$\{\sigma^0_{m}\} = [A^*]\{\varepsilon^0\} + [B^*]\{\varepsilon^f\} - \{\sigma^0_{ht}\} \qquad (7)$$
$$\{\sigma^f_{m}\} = 3[B^*]\{\varepsilon^0\} + [D^*]\{\varepsilon^f\} - \{\sigma^f_{ht}\}$$

Where :

$\{\sigma^0_{m}\} = \{N_{m}\}/h$, $\{\sigma^0_{ht}\} = \{N_{ht}\}/h$ are the in-plane laminate stress (Pa),

$\{\sigma^f_{m}\} = 6\{M_{m}\}/h^2$, $\{\sigma^f_{ht}\} = 6\{M_{ht}\}/h^2$, the flexural stresses at outer surfaces (Pa),

$\{\varepsilon^f\} = h\{k\}/2$, the flexural strain at outer surfaces (or surface strain).

The normalised laminate stiffness are also defined as a function of the ply stiffness [Q]:

$$[A^*]=\frac{1}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}}[Q]dz, \qquad [B^*]=\frac{2}{h^2}\int_{-\frac{h}{2}}^{\frac{h}{2}}[Q]zdz, \quad [D^*]=\frac{12}{h^3}\int_{-\frac{h}{2}}^{\frac{h}{2}}[Q]z^2dz$$

h = the laminate thickness.

Those normalised laminate stiffness have specific properties, independent of the thickness of the laminate, they are quite useful for easy identification of the laminate properties and direct comparison with other materials.

The fully inverted relation (8) below is very useful when the stresses are given quantities :

$$\begin{Bmatrix}\varepsilon^0\\ \varepsilon^f\end{Bmatrix}=\begin{bmatrix}a^* & \frac{b^*}{3}\\ b^* & d^*\end{bmatrix}\begin{Bmatrix}\sigma^0\\ \sigma^f\end{Bmatrix} \qquad (8)$$

where :

$$\{\sigma^0\}=\{\sigma^0_{mt}\}+\{\sigma^0_{ht}\} \quad \text{and}$$

$$\{\sigma^f\}=\{\sigma^f_{mt}\}+\{\sigma^f_{ht}\}$$

[a*], [d*] and [b*] are respectively the normalised in-plane compliance matrix, flexural compliance matrix and coupling matrix between in-plane and flexure.
Those normalised compliance matrices [a*], [b*] and [d*] (in units of 1/Pa) can be easily expressed as functions of the preceding normalised stiffness matrices [A*], [D*] and [B*].

Uncoupled laminates are such as [B*] and [b*] are equal to zero. In order to study the characterisation of composite materials, working with such materials simplifies the problem. Furthermore, these materials are widely used, hence very interesting.

III POLAR REPRESENTATION

1-Introduction :

The polar representation uses direction as a parameter. The mechanical behaviour of isotropic materials is independent of the direction, so there is no advantage in using the polar representation. On the other hand, composite materials have directionally variable elastic properties, so the study of their mechanical behaviour is simplified by polar representation. Indeed, polar representation allows direct access to the properties in each direction. Relations define the polar components versus cartesian components and inversely.

2-Definition (Kandil and Verchery [2])

If T is a 2-D second order tensor such as the stress and strain tensors [2]:

$$T = \begin{Bmatrix} T_{xx} \\ T_{yy} \\ T_{xy} \end{Bmatrix}$$

The polar coordinates of T versus the cartesian coordinates are :

$$t = (T_{xx} + T_{yy})/2 \qquad (9)$$

$$2re^{2ia} = T_{xx} - T_{yy} + 2i\, T_{xy}$$

The inversion of equation (9) gives the cartesian components of the second order tensor :

$$T_{xx} = t + r\cos 2a$$

$$T_{yy} = t - r\cos 2a \qquad (10)$$

$$T_{xy} = r\sin 2a$$

Mohr's circle is the graphical representation of these parameters

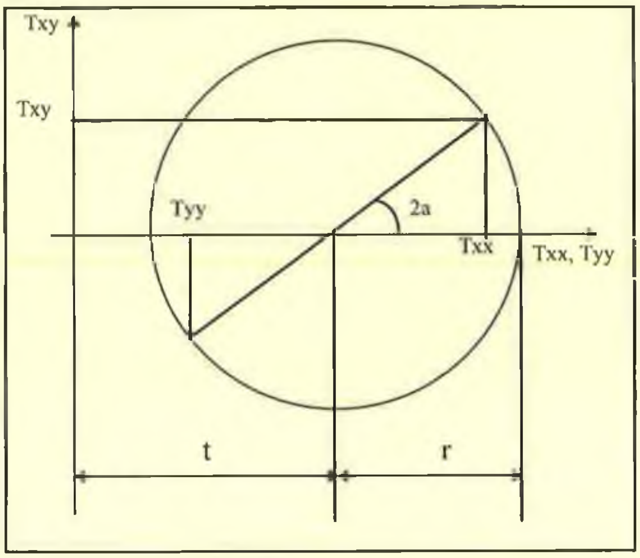


Fig. 8 - Mohr's circle

If T is an elasticity type 2-D fourth order tensor such as the in-plane stiffness and compliance laminates tensors:

$$T = \begin{bmatrix} T_{xxxx} & T_{xxyy} & T_{xxxy} \\ T_{xxyy} & T_{yyyy} & T_{xyyy} \\ T_{xxxy} & T_{xyyy} & T_{xyxy} \end{bmatrix}$$

If T_0 , T_1, $R_0e^{4i\theta_0}$, $R_1e^{2i\theta}$are the polar coordinates of T, they are calculated as follow (versus the cartesian coordinates) [2]:

$$\begin{aligned} 8T_0 &= T_{xxxx} + T_{yyyy} - 2T_{xxyy} + 4T_{xyxy} \\ 8T_1 &= T_{xxxx} + T_{yyyy} + 2T_{xxyy} \\ 8R_0e^{4i\theta_0} &= T_{xxxx} + T_{yyyy} - 2T_{xxyy} - 4T_{xyxy} + 4i(T_{xxxy} -.T_{xyyy}) \\ 8R_1e^{2i\theta_1} &= T_{xxxx} - T_{yyyy} + 2i(T_{xxxy}+T_{xyyy}) \end{aligned} \quad (11)$$

The inversion of equation (11) gives the cartesian components for the fourth order tensor :

$$
\begin{aligned}
T_{xxxx} &= T_0 + 2T_1 + R_0\cos4\theta_0 + 4R_1\cos2\theta_1 \\
T_{xxyy} &= -T_0 + 2T_1 - R_0\cos4\theta_0 \\
T_{yyyy} &= T_0 + 2T_1 + R_0\cos4\theta_0 - 4R_1\cos2\theta_1 \qquad (12) \\
T_{xyxy} &= T_0 - R_0\cos4\theta_0 \\
T_{xxxy} &= R_0\sin4\theta_0 + 2R_1\sin2\theta_1 \\
T_{xyyy} &= -R_0\sin4\theta_0 + 2R_1\sin2\theta_1
\end{aligned}
$$

3- Application to the Laminates

[Q] is a fourth order tensor, so its polar components can be calculated with the relation (11).

Kandil and Verchery [2] have introduced relations between the polar components of [Q] and the polar components of [A], [B] and [D] respectively the in-plane, coupling and bending matrices.

For in-plane stiffness matrix [A] the relations are :

$$
\begin{aligned}
T_{0A^*} &= T_{0Q} \\
T_{1A^*} &= T_{1Q} \\
R_{0A^*}e^{(4i\theta_{0A})} &= \frac{R_{0Q}}{N}\sum_{1}^{N} e^{(4i\theta_{0Qr})} \qquad (13) \\
R_{1A^*}e^{(2i\theta_{1A})} &= \frac{R_{1Q}}{N}\sum_{1}^{N} e^{(2i\theta_{1Qr})}
\end{aligned}
$$

Relation for [B]

$T_{0B} = 0$,

$T_{0D} = T_{0Q}$,

$$R_{0B*}e^{(4i\theta_{0B*})} = \frac{R_{0Q}}{N^2}\sum_{1}^{N} K_p e^{(4i\theta_{0QP})},$$

$$R_{1B*}e^{(2i\theta_{1B*})} = \frac{R_{1Q}}{N^2}\sum_{1}^{N} K_p e^{(2i\theta_{1QP})},$$

$$K_P = (Z_P^2 - Z_{P-1}^2)/h^2,$$

Relation for [D]

$T_{1B} = 0$

$T_{1D} = T_{1Q}$,

$$R_{0D*}e^{(4i\theta_{0D*})} = \frac{4R_{0Q}}{N^3}\sum_{1}^{N} K_p e^{(4i\theta_{0QP})},$$

$$R_{1D*}e^{(2i\theta_{1D*})} = \frac{4R_{1Q}}{N^3}\sum_{1}^{N} K_p e^{(2i\theta_{1QP})},$$

$$K_P = (Z_P^3 - Z_{P-1}^3)/h^3$$

K_P and h are respectively the weighting factor and the ply thickness

4-Uncoupled Laminates Properties in Polar Coordinates

The relations (13) allow the definition of new rules for composite design. For example, an uncoupled laminate has [B] = 0. Using the relation (13) between the polar components of [Q] and the ones of [A], [B] and [D], the following rule has been obtained (Kandil and Verchery [2]):

"A sufficient condition to eliminate the stretching-bending coupling is that the sum of the weighting factors of each group of orientations be equal to zero".

Example : The laminate [0/45/45/90/0/0/45] is studied.

Weighting factor for the matrix [B] :

$$K_P = (Z_P^2 - Z_{P-1}^2)/h^2$$

P is the ply number, Z the coordinate of the ply and h the ply thickness.

This laminate includes three groups of orientation : 0, 45 and 90°

There are seven plies, so Z is ranging from $[\frac{-7}{2}h;\frac{7}{2}h]$

For the direction 0° :

The first ply is located between Z=$\frac{-7}{2}h$ and $\frac{-5}{2}h$, the second between Z=$\frac{1}{2}h$ and $\frac{3}{2}h$, and the third one between Z=$\frac{3}{2}h$ and $\frac{5}{2}h$.

So the K_p sum is :

$$\sum K_p = \left\{\left(\frac{-7}{2}h\right)^2 - \left(\frac{-5}{2}h\right)^2\right\}/h^2 + \left\{\left(\frac{1}{2}h\right)^2 - \left(\frac{3}{2}h\right)^2\right\}/h^2 + \left\{\left(\frac{3}{2}h\right)^2 - \left(\frac{5}{2}h\right)^2\right\}/h^2$$

$$\sum K_p = \frac{[(-7)^2 - (-5)^2 + (1)^2 - (3)^2 + (3)^2 - (5)^2]h^2}{4h^2} = 0$$

For the direction 45°, the K_p sum is :

$$\sum K_p = \frac{[(-5)^2 - (-3)^2 + (-3)^2 - (-1)^2 + (5)^2 - (7)^2]h^2}{4h^2} = 0$$

For the direction 90°, the K_p sum is :

$$\sum K_p = \frac{[(-1)^2 + (1)^2]h^2}{4h^2} = 0$$

The sum of the weighting factors of each group of orientation is equal to zero. So, this is an uncoupled laminate.

5-Introduction of the Concepts of "Nearly" and "Quasi" :

The concepts of "nearly" and "quasi" have been introduced by Kandil and Verchery [2]. A quasi homogeneous laminate is defined as a laminate having equal stiffness matrices for in-plane and bending.

For "nearly", a norm is defined for each tensor. This norm allows to quantify the difference between two tensors.

In the same way, "Nearly isotropic" is defined by comparison between the real material and an isotropic material model approximating this real one.

To illustrate this, four examples are given for the variation of E and G as a function of the angle. For an isotropic material (Fig.9(a)), the elasticity coefficients are independent of the direction. Fig. 9(b) shows a general orthotropic laminate i.e. a material with orthogonal axis of symmetry. For a quasi-homogeneous (Fig.9(c)) the coefficients for in-plane and flexure are close. Finally, a square symmetric, quasi-homogeneous laminate (Fig.9(d)) which is an othotropic material for which the two directions are equivalent.

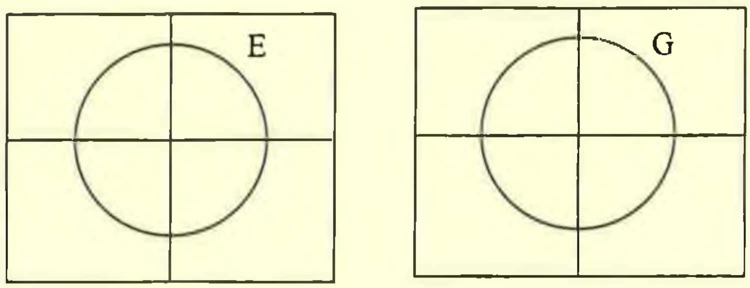


Fig. 9 (a) Variation of E and G with the angle for an Isotropic material

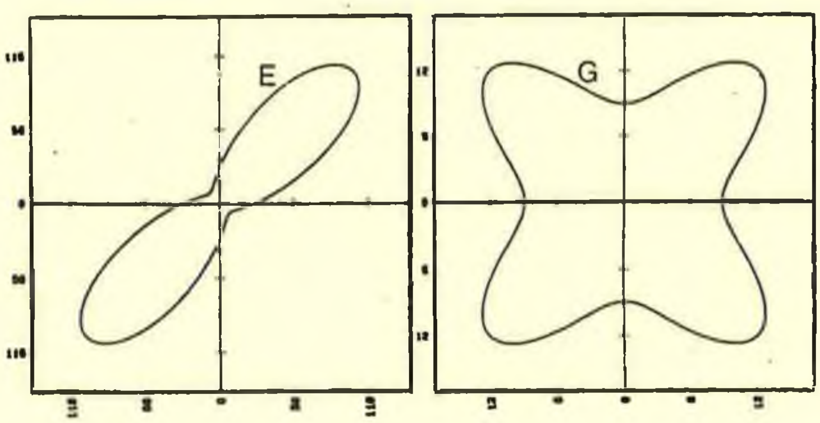


Fig. 9 (b)- Variation of E and G with the angle for a general orthotropic laminate.

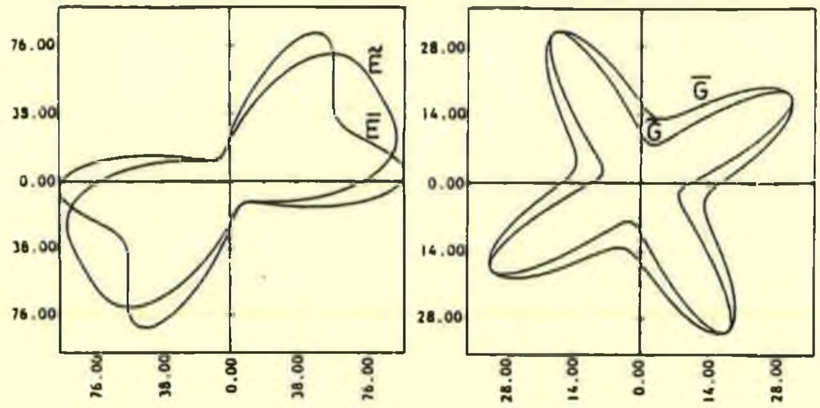


Fig. 9 (c)- Variation of E and G with the angle for a quasi-homogeneous orthotropic laminate.

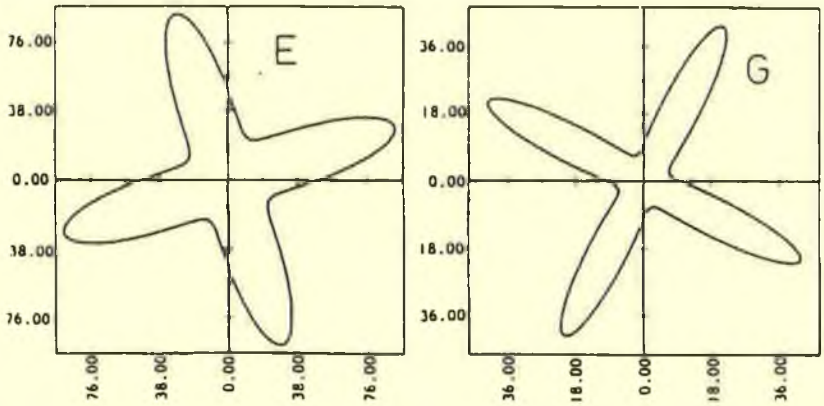


Fig. 9 (d)- Variation of E and G with the angle for a square symmetric, quasi homogeneous laminate.

IV TEST METHODS OF CHARACTERISATION [3]

Different test methods exist for the characterisation of composite materials. The type of laminates which can be tested and the properties which can be determined with each of these tests are described below. Only the in-plane characterisation is presented here because it is the only one interesting for the present work.

1-Classic Characterisation with the Tensile Test Method :

The tensile test method is used for the orthotropic laminate in their axis (A_{16}=A_{26}=0). This test allows determination of the effective modulus in loading direction and the effective laminate's Poisson ratio. The Principal axis of the laminate has to be known to characterise the material in all the directions. The test does not enable determination of the shear modulus.

2-Off-Axis Tensile Test :

The off-axis test specimen consists of a laminate with all layers at equal fibre orientation θ, with respect to the longitudinal axis of the specimen. This test method is often used to measure off-axis tensile modulus.

3-Shear Test Methods :

- the [±45] coupon test consists of the tensile test of $[45/-45/45/-45]_s$, laminate. So, the method can not be used for whatever laminate.
- The off-axis test method (seen above)
- Rail shear test method : the laminate consists of 8-12 laminae whose fibre orientation is perpendicular or parallel to the longitudinal axis of the rail.
- Torsion method : only for solid rod (unidirectional specimen only) or a tubular specimen.

Most of these test methods can only be used for particular kind of laminates and with special conditions. For instance, the classic characterisation with the tensile test method only is used for orthotropic materials in their axis and only allows determination of the effective modulus in the loading direction.

However, the tensile test method allows study of the widest range of material, but the principal axis has to be known.

V TOOLS FOR IN-PLANE STRAIN MEASUREMENTS

1-Tools and Use

Conventional electrical resistance strain gages [3]

Electrical resistance strain gages are very sensitive for measuring deformations in composite materials. Strain measurement is based on the electrical resistance change of the gage bonded to the undergoing deformation.

Liquid metal strain gages [4]:

Soft biological tissues and tire cord-rubber exhibit so little stiffness, due to their highly compliant matrix material, that conventional strain gages cannot be used to take quantitative strain measurement of them.

Geometric Moire [5]:

Displacement and strains can be determined by putting two marks on a surface, measuring the length between them, then loading the body and measuring the length again. The difference between the two lengths is displacement, and the displacement divided by the initial length is the strain. The technique is sometimes called the grid method . If the area is large it may be more convenient to take advantage of the fact that such arrays of dots or lines (called gratings), if regular, produce an interference pattern between the loaded and unloaded array. The pattern is called a moire pattern and is related to the surface displacements in an analysable way.

Birefringent coatings[3]

Birefringent or photoelastic coatings have been applied successfully to isotropic materials for several years. The method consists of bonding a thin sheet of photoelastic material to the surface of the specimen, such that the bonded interface is reflective. When the specimen is loaded, the surface strains are transmitted to the coating and produce a fringe pattern which is recorded and analysed by means of a reflection polariscope.

Holographic techniques[3]

Holography is an optical technique based on the optical interference produced by superposition of coherent light waves reflected from the object under consideration and those of a coherent reference beam. The laser is an ideal source of coherent monochromatic light.

Speckle interferometric technique[3]

Speckle interferometry makes use of the speckle pattern produced on the surface of an object illuminated by coherent light. It has many characteristics complementary to those of holographic interferometry.

Shearography[6]

Shearography is an interferometric method allowing full-field measurement of surface-displacement derivatives. The object to be tested is illuminated by a point source of coherent light. An image shearing camera produces a pair of laterally sheared images in the image plane ; hence, the method is named shearography. As the object is illuminated with coherent light, the two sheared images interfere with each other producing a random interference pattern commonly known as a speckle pattern. When the object is deformed, this speckle pattern is slightly modified. Superposition of the two speckle patterns by double exposure yields a fringe pattern depicting the derivatives of the surface displacements.

2-Strain-Gage Technology applied to Composite Materials [7]

The most common and cheapest tools used to obtain experimental results for strain measurement are strain-gages. But measurement of mechanical behaviour with composite materials is not as easy as with isotropic materials. Hence, slight gage misalignment, enhancement of transverse sensitivity errors due to anisotropic material properties and strain measurements near a free edge can lead to erroneous interpretation of experimental results. These three problems are discussed below.

Measurement Error due to Gage Misalignment

A strain-measurement error occurs whenever a strain gage (or, for that matter, any strain measurement device) is inadvertently misaligned with respect to the intended direction of strain measurement. For a single strain gage mounted in a biaxial-strain field, the magnitude of measurement error depends upon three factors :

- the magnitude of the misalignment error, β, where β equals the angle between the gage axis after bonding and the intended axis of strain measurement.
- the ratio of the algebraic maximum and minimum principal strains,
- the angle ϕ between the maximum-principal strain axis and the intended axis of strain measurement which is linked to the angle θ between the stress direction and the principal direction of the material.

These angles and their impact are shown by the figures below.

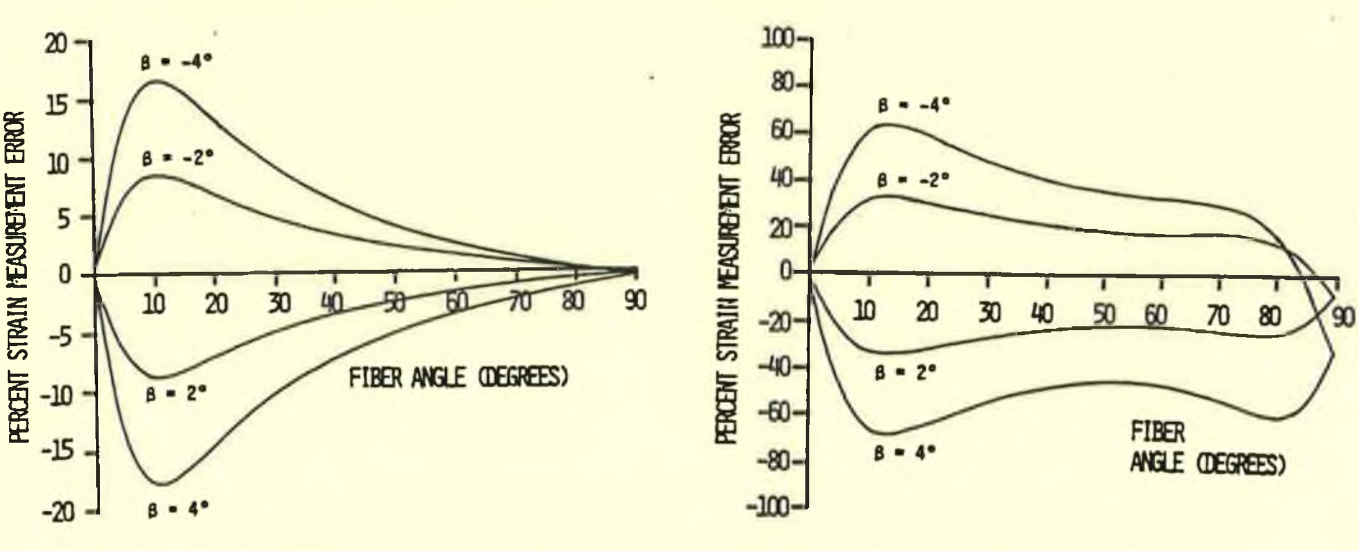


Fig. 10 Percentage error in axial (a) and transverse (b) strain measurement due to misaligned strain gage as a function of fibre axis

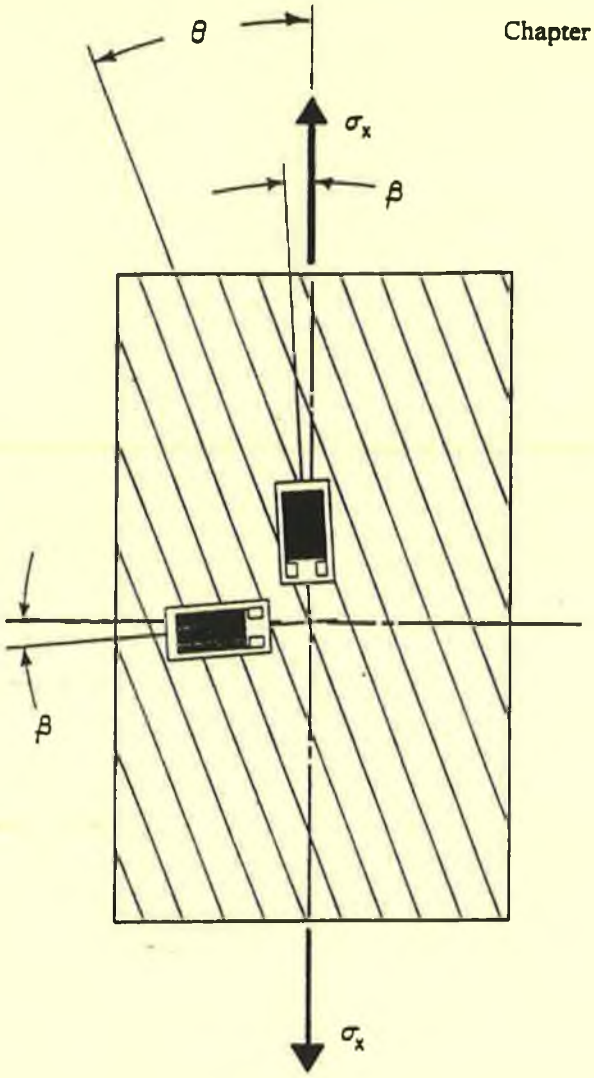


Fig. 11 A misaligned biaxial rosette mounted on an off-axis composite specimen

Transverse sensitivity errors

Errors due to gage transverse sensitivity are present in any strain-gage measurement unless :

(a) the gage is subjected to a uniaxial-stress field,

(b) the major axis of the gage is oriented parallel to the applied stress,

(c) the gage is mounted on a material whose Poisson's ratio equals $\mu_0 = 0.285$.

Now, in the general application of strain gages it is often the case that all three of these conditions are violated, and yet errors due to transverse sensitivity are often still very low. The reasons for this are that gage manufacturers have been successful in reducing the value of the transverse sensitivity coefficient to very low levels, and that Poisson's ratio for most common structural materials is relatively close to μ_0 .

However, the orthotropic nature of composites results in a propensity towards transverse-sensitivity error which would not be expected upon experience with isotropic materials. This enhancement of transverse-sensitivity effects can be traced to the fact that effective Poisson's ratio of a composite is often very much different than μ_0. The figures (12) illustrate this.

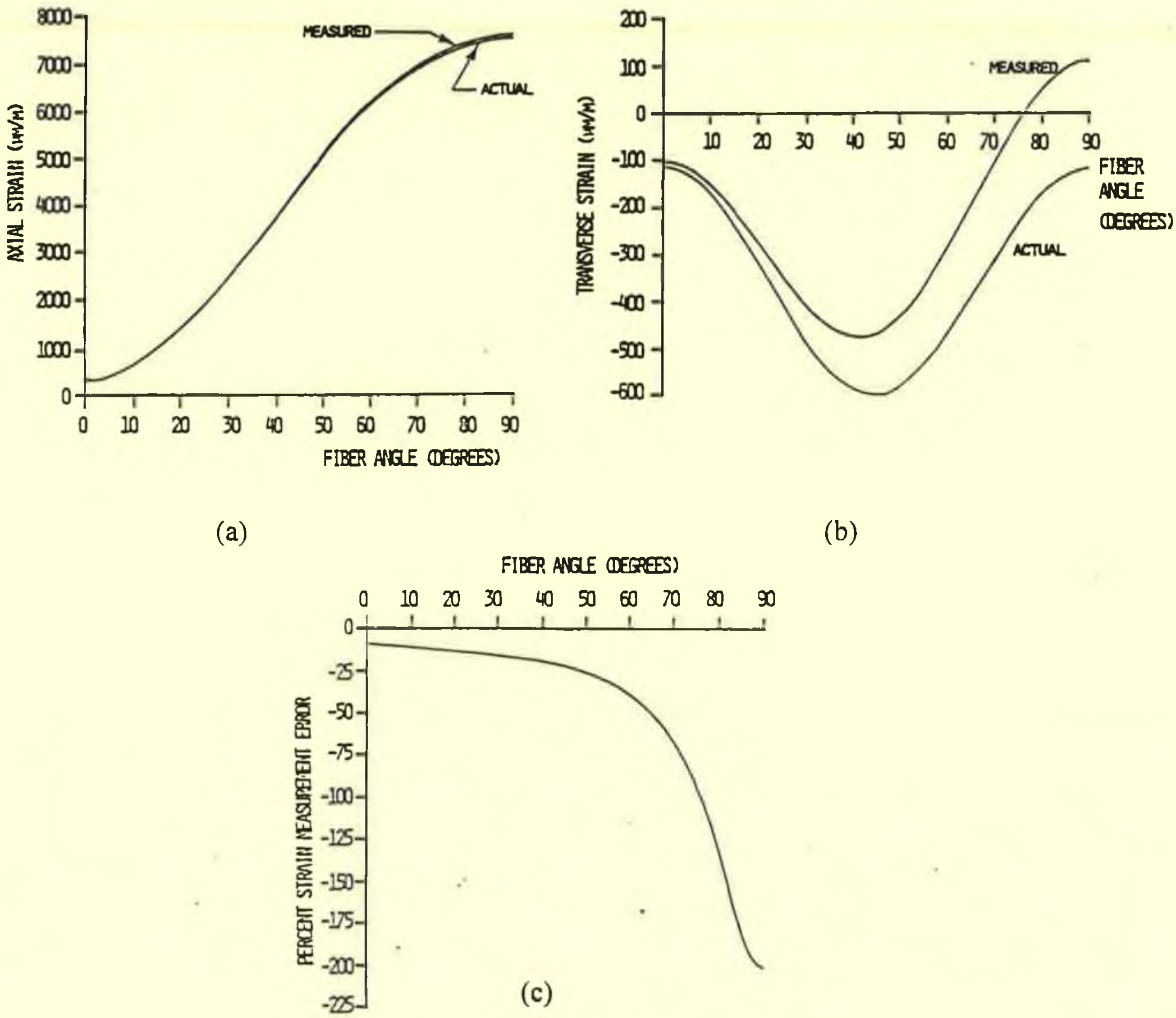


Fig. 12

(a) Actual and measured axial strain (prior to correction for transverse sensitivity effects) as a function of fibre angle.

(b) actual and measured transverse strain (prior to correction for transverse sensitivity effects) as a function of fibre angle.

(c) Percentage error in measured transverse strain (prior to correction for transverse sensitivity effects) as a function of fibre angle.

Strain measurement near a free edge

A final caution regarding strain measurement near a "free edge" is in order. The term "free edge" refers to a boundary of a composite which is not subjected to any external loading. Typical examples include the two unloaded sides of a uniaxial-tensile specimen or a cut out in a composite panel. Since most composite panels are relatively thin-plate like structures, it is appropriate to analyse composites using thin-plate theory. The assumptions above imply that out-of-plane normal and shear stresses, usually denoted σ_z, τ_{xz}, and τ_{yz}, are all zero.

These assumptions are well satisfied at regions removed from a free edge. However, near a free edge neither the plane-stress assumption nor the Kirchhoff hypothesis is valid. That is, near a free edge, significant out-of-plane stresses (σ_z, τ_{xz}, and τ_{yz}) and out-of-plane strains (ε_z, ε_{xz}, and ε_{yz}) are all induced. Near a free edge, surface strain measurements may not be related to subsurface strains. Therefore caution must exercised when applying strain gages near free edge-when attempting to measure strain concentration near cut out in a composite panel, for example.

B-STRAIN STATE

I-DETERMINATION OF THE STRAIN STATE

1-Basic Calculation of the Strain State

Strain is a 2-D second order tensor so it is represented by a Mohr circle and its polar components t, r, a are respectively the centre abscissa, the radius and the direction in which the strain is measured .

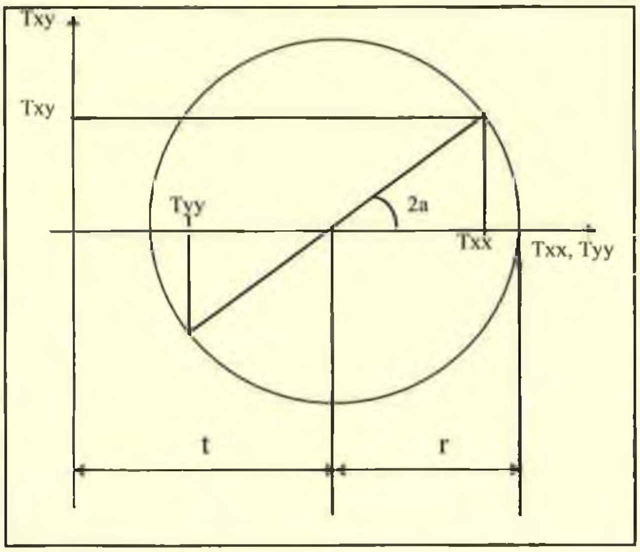


Fig. 13 Mohr's Circle

The knowledge of t, r and a is possible with three strain measurements dephased of $\pi/4$: T_0, $T_{\pi/4}$, $T_{\pi/2}$.

If t = abscissa the Mohr circle centre

r = Mohr circle radius

Then $t = (T_0 + T_{\pi/2})/2$

$$2re^{2ia} = T_0 - T_{\pi/2} + i\,(T_0 + T_{\pi/2} - 2T_{\pi/4}) \qquad (14)$$

The measurement of three strain values is enough to characterise the Mohr circle and so the strain in any direction. However, it is more accurate to measure more than three values of strain and to combine the observations to give better values of t, r and a. The least square method can be used in order to evaluate the strain state with more than three values.

2-Least Squares Method

Three points determine only one circle, but generally more than one circle will appear to fit a set of data greater than three. To avoid individual judgement in constructing the approximating circle, it is necessary to agree on a definition of a "best fitting circle".

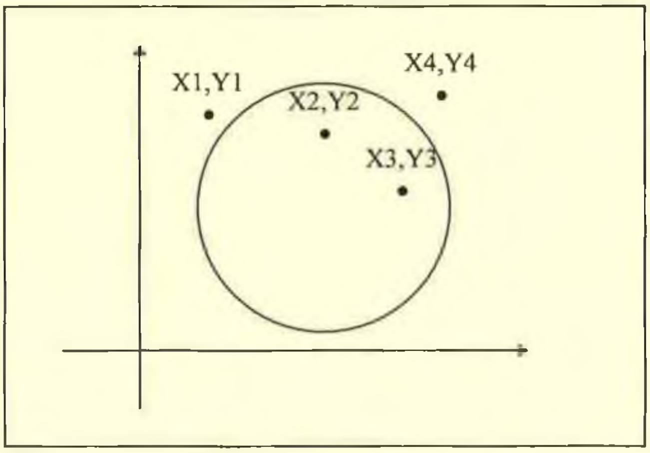


Fig. 14 Best fitting circle

Consider Fig.14 :

For a given value of x, say x_1, there will be a difference between the value y_1 and the corresponding value as determined from the circle C. We denote this difference by d_1, which is sometimes referred to as a deviation, error or residual and may be positive, negative or zero. Similarly, corresponding to the values x_2, ..., x_n, we obtain the deviations d_2,......,d_n. A measure of the "goodness of fit" of the curve C to the set of data is provided by the quantity $m=\sum_{i=1}^{n} d_i^2$. If this is small the fit is good, if it is large, the fit is bad. The best fitting circle is the one minimising m.

3-Least Squares Method Applied to the Strain Problem.

The principal direction axes are unknown, so a direction is chosen as a reference for the angles. The strain in this direction is given by $t + r \cos 2\alpha$.

The least squares method permits, with a certain number of measures, the determination of t, r et α .

We consider a sample of n measures A_i in the θ_i directions (θ is the angle between the direction and the direction of reference) : $A_i = t + r \cos 2\alpha \cos 2\theta_i - r \sin 2\alpha \sin 2\theta_i$.

For n measures, the vector 'measure' is :

$$\begin{Bmatrix} A_1 \\ A_2 \\ \\ A_n \end{Bmatrix} = \begin{bmatrix} 1 & \cos 2\theta_1 & -\sin 2\theta_1 \\ 1 & \cos 2\theta_2 & -\sin 2\theta_2 \\ & & \\ 1 & \cos 2\theta n & -\sin 2\theta n \end{bmatrix} . \begin{Bmatrix} t \\ r\cos 2\alpha \\ r \sin 2\alpha \end{Bmatrix} \qquad A = \Theta\,\varepsilon \qquad (15)$$

Θ and A are known because the angles are chosen and A is the result of the tests. For three measurements Θ is square and, in general, non-singular matrix ; thus, Θ can be inversed and ε becomes :

$$\varepsilon = \Theta^{-1} A$$

For more measures, the problem is more complicated because Θ is not square anymore. Assuming that θ_{ij} are the components of Θ and z_i the components of ε. The deviations are given by :

$$\begin{aligned} d_1 &= \theta_{11} z_1 + \theta_{12} z_2 + \theta_{13} z_3 - A_1 \\ d_2 &= \theta_{21} z_1 + \theta_{22} z_2 + \theta_{23} z_3 - A_2 \\ &\dots\dots\dots\dots\dots\dots \\ d_n &= \theta_{n1} z_1 + \theta_{n2} z_2 + \theta_{n3} z_3 - A_n \end{aligned} \qquad (16)$$

Equations may be written :

$$d_i = \theta_{ij} z_j - A_i \text{ or } d = \Theta\varepsilon - A.$$

The "best fitting circle" is the one which minimises $d_i.d_i$ so we have

$$d_i \frac{\partial d_i}{\partial z_j} = 0 \qquad \text{or} \quad \frac{\partial d_i}{\partial z_j} = \theta_{ij}$$

$$\text{so, } d_i\theta_{ij} = 0 \qquad (17)$$

Eliminating d_i from (17)

$$(\theta_{ij}\, z_j - A_i)\, \theta_{ij} = 0 \qquad (18)$$

so

$$\theta_{ji}(\theta_{ij}\, z_j - A_i) = 0$$

or

$$\Theta^T\Theta\varepsilon - \Theta^t A = 0 \qquad (19)$$

Θ^T is the transpose matrix of Θ.

$(\Theta^T\Theta)$ is a square matrix - schematically, if Θ is a (q x p) matrix, Θ^T is a (p x p) matrix and then, $(\Theta^T\Theta)$ is a (q x p) x (p x q) = (q x q) matrix- and has reciprocal (if the determinant is not 0). Hence, multiplying before each term by $(\Theta^T\Theta)^{-1}$, we obtain

$$\varepsilon = (\Theta^T\, \Theta)^{-1}\Theta^t A \qquad (20)$$

The solution of this system allows the evaluation of t et r and then the principal directions owing to the use of the Mohr circle.

The deviation is evaluated with the following relation.

$$d = \{(\Theta^t\, \Theta)^{-1}\Theta^T - I\}A \qquad (21)$$

II-STUDY OF THE EXPERIMENT WITH FOUR GAGES

1-Interest in Using Four Gages

The more gages that are used the better is the approximation . But, the more gages that are added the more expensive is the measurement. Hence, the advantages and disadvantages of a fourth gage are studied here.

Firstly, the difference between three and four gages is studied. A strain state (t, r, α) is defined and the strain in four directions is calculated (with the Mohr circle).

Then, to each strain a random error is added , the strain state defined by the three first ones is calculated, and then the strain state defined by the four gages is calculated using the Least Squares Method.

In appendix A, 100 samples of 4 random errors are determined. The three first ones of each sample are applied to the three gages and the whole sample to the four gages. Thus, the impact of the fourth gage can be measured. The following percentages in favour of each case are obtained.

Best approximation	Three gages	Four gages	Same approximation
Value of r	17%	16%	67%
Value of t	27%	54%	19%

Thus, the table shows that the fourth gage gives a better accuracy more often than the three gages.

2-Position of the Gages

Having a method to determine the optimum Mohr circle from a sample of measures Ai, and knowing that a fourth gage is more accurate, the angles which minimise the impact of the errors on the Mohr circle are now studied. 8 samples of four gages are used (the reference for the angles is the principal strain direction) :

Series 1 : $(0,2\pi/3, 4\pi/3, \pi/4)+\pi/4$ (out of phase by $\pi/4$)

Series 2 : $0, 2\pi/3, 4\pi/3, \pi/4$

Series 3 : $(0, 2\pi/3, 4\pi/3, \pi/2)+\pi/4$

Series 4 : $0, 2\pi/3, 4\pi/3, \pi/2$

Series 5 : $(0, \pi/4, \pi/2, 3\pi/4)+\pi/8$

Series 6 : $0, \pi/4, \pi/2, 3\pi/4$

Series 7 : $(0, \pi/4, \pi/2, 3\pi/4)+\pi/4$

Series 8 : $(0, \pi/4, \pi/2, 3\pi/4)+\pi/3$

Taking $t = 24$ and $r = 12$, the strain in each direction is calculated and all the combinations of 10% errors are applied to each of the samples of four gages. The study is done in appendix B and summed up by the figures next page. We note that the four gages $0, \pi/4, \pi/2, 3\pi/4$ gives the best results

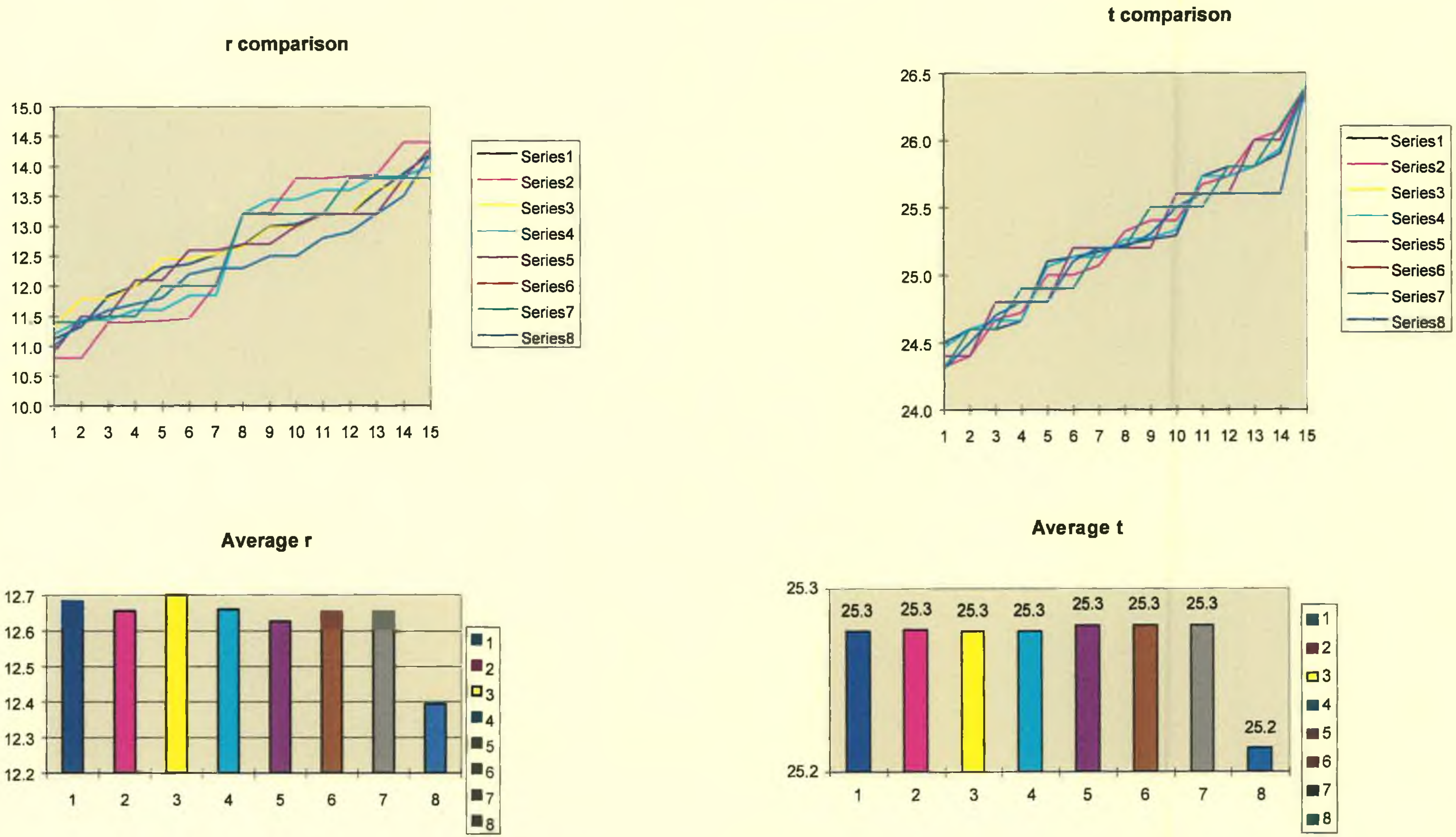


Fig. 15 Comparison of the impact of 10% errors on r and t calculated

Furthermore, the study shows that the best results are obtained with the first gage in the principal strain direction (series 6).

Thus, a first measure could determine the principal strain direction and then the first gage could be applied in this direction (Fig.16).

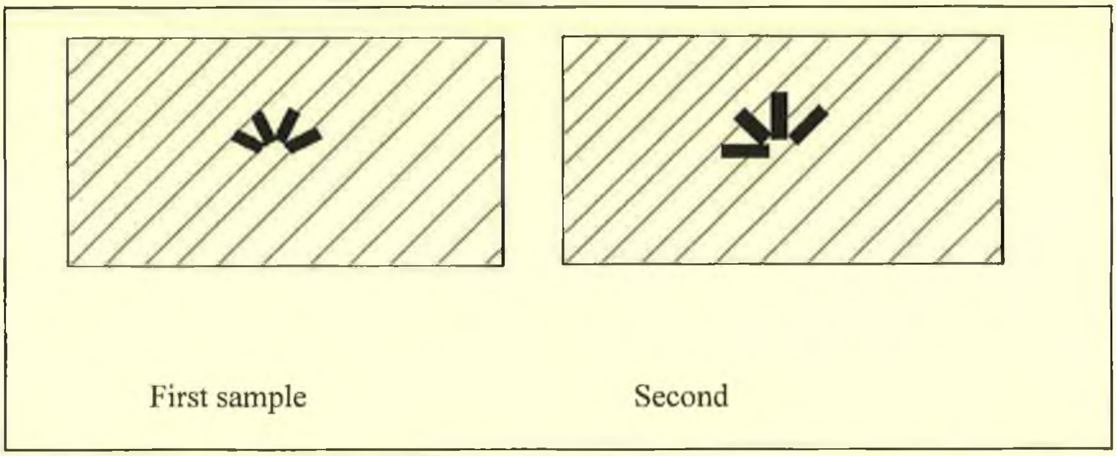


Fig. 16. Best direction to apply the gages

Finally, the study of M.E Tuttle [7] (see chapter A-V-2) shows, for an unidirectional, that the stress applied in the principal directions of the material permits the minimisation of the error due to the gages. Hence, if the direction in which the sample is cut can be chosen, a first sample can be used to determine the principal direction of the material and then a second one, where the stress is applied in this direction.

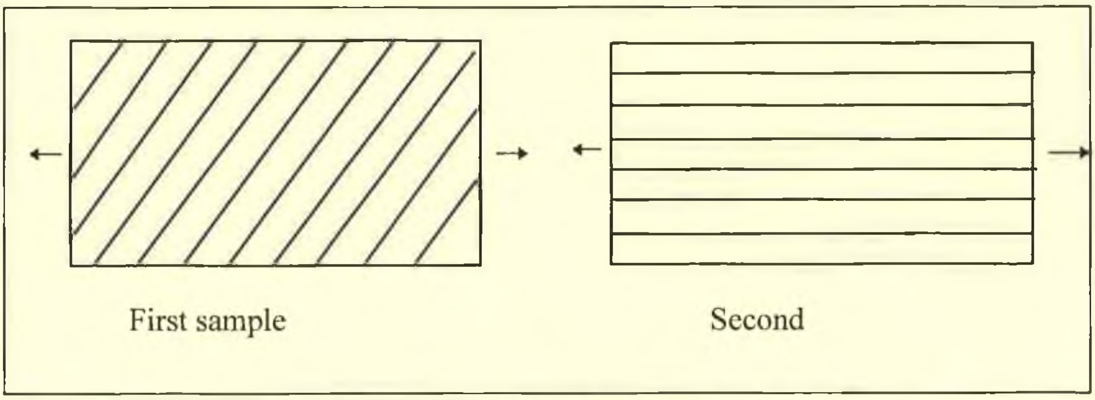


Fig. 17 The stress applied in the principal direction gives better results

3-Error Calculation

Even if the method permits the optimisation of the values, a certain amount of errors exists. Some of them are due to the tools used to measure and may sometimes be identified. The aim of the following part is the determination of their impact on the final results in order to measure its accuracy.

If :

A : the measure,

ΔA : measurement error,

Δθ : error on the angle between the gage and the reference gage,

Δx : absolute error associated to a measure,

Δy :relative error associated to a measure,

$A = t + r\cos(2\theta)$

$A+\Delta A = [t + r\cos(2\theta + \Delta\theta)]\,(1+\Delta y) + \Delta x$

$A+\Delta A = [t + r\cos(2\theta)\cos(2\Delta\theta) - r\sin(2\theta)\sin(2\Delta\theta)]\,(1+\Delta y) + \Delta x$

$A+\Delta A = t + r\cos(2\theta) + [t + r\cos(2\theta)]\,\Delta y - r\sin(2\theta)\,2\Delta\theta + \Delta x$

$$\Delta A = [t + r\cos(2\theta)]\,\Delta y - r\sin(2\theta)\,2\Delta\theta + \Delta x \qquad (22)$$

Taking four measures A_i at four angles dephased of π/4, using (22)

$$\begin{aligned} A_1 &= t + r\cos(2\theta) + [t + r\cos(2\theta)]\,\Delta y_1 + \Delta x_1 \\ A_2 &= t - r\sin(2\theta) + [t - r\sin(2\theta)]\,\Delta y_2 - r\cos(2\theta)\,2\Delta\theta_2 + \Delta x_2 \qquad (23) \\ A_3 &= t - r\cos(2\theta) + [t - r\cos(2\theta)]\,\Delta y_3 + r\sin(2\theta)\,2\Delta\theta_3 + \Delta x_3 \\ A_4 &= t + r\sin(2\theta) + [t + r\sin(2\theta)]\,\Delta y_4 + r\cos(2\theta)\,2\Delta\theta_4 + \Delta x_4 \end{aligned}$$

Using the Least squares method for 0, π/4, π/2, 3π/4, the solution is $\varepsilon = (\Theta^T\Theta)^{-1}\Theta^t A$ or :

$$\begin{Bmatrix} t_m \\ r_m\cos(2\theta_m) \\ r_m\sin(2\theta_m) \end{Bmatrix} = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.5 & 0 & -0.5 & 0 \\ 0 & -0.5 & 0 & 0.5 \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix}$$

$$t_m = t + 0.25\, t\, (\Delta y_1 + \Delta y_2 + \Delta y_3 + \Delta y_4) + 0.25\, (\Delta x_1 + \Delta x_2 + \Delta x_3 + \Delta x_4)$$
$$+ 0.5\, (\Delta\theta_4 - \Delta\theta_2)\, r \cos(2\theta) + 0.5\, \Delta\theta_3\, r \sin(2\theta) \qquad (24)$$
$$+ 0.25\, (\Delta y_1 - \Delta y_3)\, r \cos(2\theta) + 0.25\, (\Delta y_4 - \Delta y_2)\, r \sin(2\theta)$$

$$r_m \cos(2\theta_m) = r \cos(2\theta) + 0.5\, t\, (\Delta y_1 - \Delta y_3) + 0.5\, (\Delta x_1 - \Delta x_3)$$
$$+ 0.5\, (\Delta y_1 + \Delta y_3)\, r \cos(2\theta) - \Delta\theta_3\, r \sin(2\theta) \qquad (25)$$

$$r_m \sin(2\theta_m) = r \sin(2\theta) + 0.5\, t\, (\Delta y_4 - \Delta y_2) + 0.5\, (\Delta x_4 - \Delta x_2)$$
$$+ 0.5\, (\Delta y_4 + \Delta y_2)\, r \sin(2\theta)$$
$$+ (\Delta\theta_4 + \Delta\theta_2)\, r \cos(2\theta) \qquad (26)$$

θ is the angle between the principal direction and the direction of the gage chosen as a reference.

Relations (24), (25) and (26) shows that in order to calculate the error, we need r, t and θ which are the values searched. However, the study in appendix C shows that, using the values determined with the Least Squares Method, the errors obtained are quite close from the errors obtained with the chosen values (r=12 and t=24) . Thus, knowing the accuracy of the measurement, the accuracy of the results can be evaluated.

The aim of this chapter was to show the interest in increasing the number of gages in order to give a better approximation of the state strain. The accuracy has been evaluated but in order to determine the mechanical behaviour of the material, the impact of the error on the final results must be calculated.

C-MECHANICAL BEHAVIOUR

The study has been limited to uncoupled materials. The mechanical behaviour of laminates is defined by the relation (8) :

$$\begin{Bmatrix} \varepsilon^0 \\ \varepsilon^f \end{Bmatrix} = \begin{bmatrix} a* & \frac{b*}{3} \\ b* & d* \end{bmatrix} \begin{Bmatrix} \sigma^0 \\ \sigma^f \end{Bmatrix}$$

An uncoupled material is such as b*=0, so the relation for an in-plane loading becomes :

$\{\varepsilon\} = [a*]\{\sigma\}$

Hence, the mechanical behaviour is much easier to study.

In order to determine the mechanical behaviour of composite materials, we have already shown that the polar coordinates are interesting due to the character anisotropic of these materials. The polar coordinates allow a direct access to the elasticity coefficients in any direction. However, all the relations concerning the mechanical behaviour of composites are defined with respect of cartesian coordinates system. To complete this work, it is necessary to know their equivalent in polar coordinates.

I-STRESS-STRAIN RELATIONS

The first step is to define the stress-strain relations for the ply such as :

$\{\sigma_p\} = [Q_p]\{\varepsilon_p\}$ or its inverse $\{\varepsilon_p\} = [S_p]\{\sigma_p\}$ (p stands for polar).

Introducing the cartesian coordinates of ε and Q versus their polar counterparts in the relation $\{\sigma_c\} = [Q_c]\{\varepsilon_c\}$, the cartesian coordinates of σ versus the polar ones of Q and ε are defined. Then, introducing the cartesian coordinates of σ versus the polar ones of Q and ε in the relation giving the polar coordinates of σ versus its cartesian ones, the relation becomes strictly polar. Finally, the terms are organised to obtain a relation such as $\{\sigma_p\} = [Q_p]\{\varepsilon_p\}$.

The same operation can be completed for the laminates.

1- Relationship Between Cartesian and Polar Forms of ε

ε is a 2-D second order tensor so it can be defined as follow :

$$\begin{aligned} \varepsilon_{11} &= t_\varepsilon + r_\varepsilon \cos 2a_\varepsilon \\ \varepsilon_{22} &= t_\varepsilon - r_\varepsilon \cos 2a_\varepsilon \\ 2\varepsilon_{12} &= 2r_\varepsilon \sin 2a_\varepsilon = \varepsilon_{16} \end{aligned} \tag{27}$$

2- Relationship Between Cartesian and Polar Forms of Q

Q_c (c stands for cartesian) is a 2-D four order tensor so it can be defined as follow :

$$Q_c = \begin{bmatrix} Q_{c11} & Q_{c12} & Q_{c16} \\ Q_{c12} & Q_{c22} & Q_{c26} \\ Q_{c16} & Q_{c26} & Q_{c66} \end{bmatrix}$$

$$\begin{aligned} Q_{c\,11} &= T_0 + 2T_1 + R_0 \cos 4\theta_0 + 4R_1 \cos 2\theta_1 \\ Q_{c\,12} &= -T_0 + 2T_1 - R_0 \cos 4\theta_0 \\ Q_{c\,22} &= T_0 + 2T_1 + R_0 \cos 4\theta_0 - 4R_1 \cos 2\theta_1 \\ Q_{c\,66} &= T_0 - R_0 \cos 4\theta_0 \\ Q_{c\,16} &= R_0 \sin 4\theta_0 + 2R_1 \sin 2\theta_1 \\ Q_{c\,62} &= -R_0 \sin 4\theta_0 + 2R_1 \sin 2\theta_1 \end{aligned} \tag{28}$$

3-Cartesian Coordinates of σ Versus the Polar Ones of Q and ε

The stress-strain relation in cartesian co-ordinates is $\sigma_c = [\, Q_c \,]\, \varepsilon_c$ so

This relation gives the cartesian coordinates of σ versus the ones of Q and ε

$$\begin{aligned} \sigma_{11} &= Q_{c\,11}\, \varepsilon_{11} + Q_{c\,12}\, \varepsilon_{22} + Q_{c\,16}\, \varepsilon_{16} \\ \sigma_{22} &= Q_{c\,12}\, \varepsilon_{11} + Q_{c\,22}\, \varepsilon_{22} + Q_{c\,26}\, \varepsilon_{16} \\ \sigma_{12} &= Q_{c\,16}\, \varepsilon_{11} + Q_{c\,26}\, \varepsilon_{22} + Q_{c\,66}\, \varepsilon_{16} \end{aligned} \tag{29}$$

Then, the polar coordinates of Q and ε are introduced on the right side of the relation:

$\sigma_{11} = Q_{c\,11}\, \varepsilon_{11} + Q_{c\,12}\, \varepsilon_{22} + Q_{c\,16}\, \varepsilon_{16}$

Where :

$Q_{c\,11} . \varepsilon_{11} = (T_0 + 2T_1 + R_0 \cos 4\theta_0 + 4 R_1 \cos 2\theta_1)\,(t_\varepsilon + r_\varepsilon \cos 2a_\varepsilon)$

$Q_{c\,12}\, \varepsilon_{22} = (-T_0 + 2T_1 - R_0 \cos 4\theta_0)\,(t_\varepsilon - r_\varepsilon \cos 2a_\varepsilon)$

$2\, Q_{c\,16}\, \varepsilon_{12} = 2\,(R_0 \sin 4\theta_0 + 2 R_1 \sin 2\theta_1)\,(r_\varepsilon \sin 2a_\varepsilon)$

The same operation can be done for σ_{22} and σ_{12}

$\sigma_{22} = Q_{c\,12}\, \varepsilon_{11} + Q_{c\,22}\, \varepsilon_{22} + Q_{c\,26}\, \varepsilon_{16}$

Where :

$Q_{c\,12} . \varepsilon_{11} = (-T_0 + 2T_1 - R_0 \cos 4\theta_0)\,(t_\varepsilon + r_\varepsilon \cos 2a_\varepsilon)$

$Q_{c\,22}\, \varepsilon_{22} = (T_0 + 2T_1 + R_0 \cos 4\theta_0 - 4R_1 \cos 2\theta_1)\,(t_\varepsilon - r_\varepsilon \cos 2a_\varepsilon)$

$2\, Q_{c\,26}\, \varepsilon_{12} = 2\,(-R0 \sin 4\theta 0 + 2\, R1 \sin 2\theta 1)\,(r\varepsilon \sin 2a\varepsilon)$

$\sigma_{12} = Q_{c\,16}\, \varepsilon_{11} + Q_{c\,26}\, \varepsilon_{22} + Q_{c\,66}\, \varepsilon_{16}$

Where :

$Q_{c\,16} . \varepsilon_{11} = (R_0 \sin 4\theta_0 + 2 R_1 \sin 2\theta_1)(t_\varepsilon + r_\varepsilon \cos 2a_\varepsilon)$

$Q_{c\,26}\, \varepsilon_{22} = (-R_0 \sin 4\theta_0 + 2 R_1 \sin 2\theta_1)(t_\varepsilon - r_\varepsilon \cos 2a_\varepsilon)$

$2\, Q_{c\,66}\, \varepsilon_{12} = 2\,(T_0 - R_0 \cos 4\theta_0)(r_\varepsilon \sin 2a_\varepsilon)$

4-Stress - Strain Relation in Polar

4.1 Using the Stiffness

σ_c is a 2-D second order tensor so it can be defined as follow :

$t_\sigma = (\sigma_{11} + \sigma_{22})/2$

$2r_\sigma e^{2ia\sigma} = \sigma_{11} - \sigma_{22} + 2i\sigma_{12}$ (30)

So, using the relations (29) and (30), t_σ is :

$t_\sigma = 4T_1 t_\varepsilon + 4R_1 \cos 2\theta_1\, r_\varepsilon \cos 2a_\varepsilon + 4\, R_1 \sin 2\theta_1\, r_\varepsilon \sin 2a_\varepsilon$ (31)

$r_\sigma \cos 2a_\sigma$ is :

$$r_\sigma \cos 2a_\sigma = 4\, R_1 \cos 2\theta_1\, t_\varepsilon + (2\, R_0 \cos 4\theta_0 + 2\, T_0)\, r_\varepsilon \cos 2a_\varepsilon + 2\, R_0 \sin 4\theta_0\, r_\varepsilon \sin 2a_\varepsilon \qquad (32)$$

$r_\sigma \sin 2a_\sigma$ is :

$$r_\sigma \sin 2a_\sigma = 4R_1 \sin 2\theta_1\, t_\varepsilon + (2\, R_0 \sin 4\theta_0)\, r_\varepsilon \cos 2a_\varepsilon + (2\, T_0 - 2\, R_0 \cos 4\theta_0)\, r_\varepsilon \sin 2a_\varepsilon \qquad (33)$$

So, using the stiffness, for an unidirectional composite, the stress strain relation in polar coordinates is the following :

$$t_\sigma = 4[t_\varepsilon\, T_{1Q} + r_\varepsilon\, R_{1Q} \cos 2(\theta_{1Q} - a_\varepsilon)]$$

$$r_\sigma e^{2ia_\sigma} = 4(R_{1Q} e^{2i\theta_{1Q}})\, t_\varepsilon + 2T_{0Q}\,(r_\varepsilon e^{2ia_\varepsilon}) + 2(R_{0Q} e^{4i\theta_{0Q}}\, r_\varepsilon e^{-2ia_\varepsilon}) \qquad (34)$$

We introduce the following relation :

$\{\sigma_p\} = [Q_p]\,\{\varepsilon_p\}$

Where :

$$\sigma_p = \begin{Bmatrix} t_\sigma \\ r_\sigma \cos 2a_\sigma \\ r_\sigma \sin 2a\sigma \end{Bmatrix}, \qquad \varepsilon_p = \begin{Bmatrix} t_\varepsilon \\ r_\varepsilon \cos 2a_\varepsilon \\ r_\varepsilon \sin 2a_\varepsilon \end{Bmatrix}$$

$$[Q_p] = \begin{bmatrix} 4T_{1Q} & 4R_{1Q}\cos 2\theta_{1Q} & 4R_{1Q}\sin 2\theta_{1Q} \\ 4R_{1Q}\cos 2\theta_{1Q} & 2T_{0Q} + 2R_{0Q}\cos 4\theta_{0Q} & 2R_{0Q}\sin 4\theta_{0Q} \\ 4R_{1Q}\sin 2\theta_{1Q} & 2R_{0Q}\sin 4\theta_{0Q} & 2T_{0Q} - 2R_{0Q}\cos 4\theta_{0Q} \end{bmatrix} \qquad (35)$$

4.2-Relation Using the Compliance

For the compliance the same kind of relation can be obtained by the same way :

$$t_\varepsilon = 4[t_\sigma\, T_{1S} + r_\sigma\, R_{1S} \cos 2(\theta_{1S} - a_\sigma)]$$

$$r_\varepsilon e^{2ia_\varepsilon} = 4(R_{1S} e^{2i\theta_{1S}})\, t_\sigma + 2T_{0S}\,(r_\sigma e^{2ia_\sigma}) + 2(R_{0S} e^{4i\theta_{0S}}\, r_\sigma e^{-2ia_\sigma}) \qquad (36)$$

We introduce the following relation :

$\{\varepsilon_p\} = [S_p]\,\{\sigma_p\}$ (37)

Where :

$$[S_p]=\begin{bmatrix} 4T_{1S} & 4R_{1S}\cos 2\theta_{1S} & 4R_{1S}\sin 2\theta_{1S} \\ 4R_{1S}\cos 2\theta_{1S} & 2T_{0S}+2R_{0S}\cos 4\theta_{0S} & 2R_{0S}\sin 4\theta_{0S} \\ 4R_{1S}\sin 2\theta_{1S} & 2R_{0S}\sin 4\theta_{0S} & 2T_{0S}-2R_{0S}\cos 4\theta_{0S} \end{bmatrix} \quad (38)$$

5-Relations for the Laminates

For an uncoupled laminate, [b*]=0 and if we consider an in-plane stress, $\{\varepsilon^f\}=0$ so the stress-strain relation is defined by :

$\{\varepsilon\} = [a^*]\,\{\sigma\}$ With : $[a^*] = [A^*]^{-1}$

a* is a 2-D fourth order tensor so it can be represented by polar components. So, the stress - strain relation in polar is obtained as before and is :

$$t_\varepsilon = 4[t_\sigma\, T_{1a^*} + r_\sigma\, R_{1a^*} \cos 2(\theta_{1a^*}-a_\sigma)]$$

$$r_\varepsilon e^{2ia_\varepsilon} = 4(R_{1a^*}e^{2i\theta_{1a^*}}).t_\sigma + 2T_{0a^*}(r_\sigma e^{2ia_\sigma}) + 2(R_{0a^*}e^{4i\theta_{0a^*}}\, r_\sigma e^{-2ia_\sigma}) \quad (39)$$

The equation (39) can be separated via the real and imaginary coefficients :

$$r_\varepsilon \cos(2a_\varepsilon) = 4\, R_{1a} \cos(2\theta_{1a}).\, t_\sigma + 2T_{0a}\, r_\sigma \cos(2a_\sigma) + 2R_{0a}\, r_{\sigma^*} \cos(4i\theta_{0a} - a_\sigma)$$

$$r_\varepsilon \sin(2a_\varepsilon) = 4\, R_{1a} \sin(2\theta_{1a}).\, t_\sigma + 2T_{0a}\, r_\sigma \sin(2a_\sigma) + 2R_{0a}\, r_{\sigma^*} \sin(4i\theta_{0a} - a_\sigma) \quad (40)$$

For the stiffness, the same relations can be determined

$$t_\sigma = 4[t_\varepsilon\, T_{1A^*} + r_\varepsilon\, R_{1A^*} \cos 2(\theta_{1A^*}-a_\varepsilon)] \quad (41)$$

$$r_\sigma e^{2ia_\sigma} = 4(R_{1A^*}e^{2i\theta_{1a^*}}).t_\sigma + 2T_{0A^*}(r_\sigma e^{2ia_\varepsilon}) + 2(R_{0A^*}e^{4i\theta_{0A^*}}\, r_\sigma e^{-2ia_\varepsilon})$$

We introduce the following relation :

$$\{\varepsilon_p\} = [S_p]\,\{\sigma_p\} \tag{37}$$

Where :

$$[S_p] = \begin{bmatrix} 4T_{1S} & 4R_{1S}\cos 2\theta_{1S} & 4R_{1S}\sin 2\theta_{1S} \\ 4R_{1S}\cos 2\theta_{1S} & 2T_{0S} + 2R_{0S}\cos 4\theta_{0S} & 2R_{0S}\sin 4\theta_{0S} \\ 4R_{1S}\sin 2\theta_{1S} & 2R_{0S}\sin 4\theta_{0S} & 2T_{0S} - 2R_{0S}\cos 4\theta_{0S} \end{bmatrix} \tag{38}$$

5-Relations for the Laminates

For an uncoupled laminate, [b*]=0 and if we consider an in-plane stress, $\{\varepsilon^f\}=0$ so the stress-strain relation is defined by :

$\{\varepsilon\} = [a^*]\,\{\sigma\}$ With : $[a^*] = [A^*]^{-1}$

a* is a 2-D fourth order tensor so it can be represented by polar components. So, the stress - strain relation in polar is obtained as before and is :

$$t_\varepsilon = 4[t_\sigma\, T_{1a*} + r_\sigma\, R_{1a*}\cos 2(\theta_{1a*} - a_\sigma)]$$

$$r_\varepsilon e^{2ia_\varepsilon} = 4(R_{1a*}e^{2i\theta_{1a*}}).t_\sigma + 2T_{0a*}(r_\sigma e^{2ia_\sigma}) + 2(R_{0a*}e^{4i\theta_{0a*}}\, r_\sigma e^{-2ia_\sigma}) \tag{39}$$

The equation (39) can be separated via the real and imaginary coefficients :

$$r_\varepsilon \cos(2a_\varepsilon) = 4\, R_{1a}\cos(2\theta_{1a}).\, t_\sigma + 2T_{0a}\, r_\sigma \cos(2a_\sigma) + 2R_{0a}\, r_{\sigma*}\cos(4i\theta_{0a} - a_\sigma)$$

$$r_\varepsilon \sin(2a_\varepsilon) = 4\, R_{1a}\sin(2\theta_{1a}).\, t_\sigma + 2T_{0a}\, r_\sigma \sin(2a_\sigma) + 2R_{0a}\, r_{\sigma*}\sin(4i\theta_{0a} - a_\sigma) \tag{40}$$

For the stiffness, the same relations can be determined

$$t_\sigma = 4[t_\varepsilon\, T_{1A*} + r_\varepsilon\, R_{1A*}\cos 2(\theta_{1A*} - a_\varepsilon)] \tag{41}$$

$$r_\sigma e^{2ia_\sigma} = 4(R_{1A*}e^{2i\theta_{1a*}}).t_\sigma + 2T_{0A*}(r_\sigma e^{2ia}\varepsilon) + 2(R_{0A*}e^{4i\theta_{0A*}}\, r_\sigma e^{-2ia}\varepsilon)$$

II-RELATION BETWEEN THE COMPLIANCE AND STIFFNESS POLAR COMPONENTS

As for the cartesian matrices $[S_p] = [Q_p]^{-1}$,(p stands for polar) the determinant must be calculated first in order to find the relation between the polar components of the two matrices.

1-Determination of the Determinant of $[Q_p]$ in Polar :

The components of the matrix $[Q_p]$ in polar are determined by equation (35) as :

$$[Q_p] = \begin{bmatrix} 4T_{1Q} & 4R_{1Q}\cos 2\theta_{1Q} & 4R_{1Q}\sin 2\theta_{1Q} \\ 4R_{1Q}\cos 2\theta_{1Q} & 2T_{0Q} + 2R_{0Q}\cos 4\theta_{0Q} & 2R_{0Q}\sin 4\theta_{0Q} \\ 4R_{1Q}\sin 2\theta_{1Q} & 2R_{0Q}\sin 4\theta_{0Q} & 2T_{0Q} - 2R_{0Q}\cos 4\theta_{0Q} \end{bmatrix}$$

Q_p is a matrix 3x3 and its determinant is given by :

$$|Q_p| = 4T_1 (4T_0^2 - 4 R_0^2 \cos^2 4\theta_0) - 4 T_1 (4R_0^2 \sin^2 4\theta_0) - 16 R_1^2 \cos^2 2\theta_1 (2T_0 - 2 R_0 \cos 4\theta_0) + 4 R_1 \cos 2\theta_1\, 4 R_1 \sin 2\theta_1\, 2 R_0 \sin 4\theta_0 + 4 R_1 \cos 2\theta_1\, 4 R_1 \sin 2\theta_1\, 2 R_0 \sin 4\theta_0 - 16 R_1^2 \sin^2 2\theta_1 (2T_0 + 2R_0 \cos 4\theta_0)$$

$$|Q_p| = 16 T_1 T_0^2 - 16 T_1 R_0^2 (\cos^2 4\theta_0 + \sin^2 4\theta_0) - 32 T_0 R_1^2 (\cos^2 2\theta_1 + \sin^2 2\theta_1) + 64 R_1^2 R_0 \sin 2\theta_1 \cos 2\theta_1 \sin 4\theta_0 + 32 R_1^2 R_0 \cos 4\theta_0 (\cos^2 2\theta_1 - \sin^2 2\theta_1)$$

$$|Q_p| = 16 T_1 T_0^2 - 16 T_1 R_0^2 - 32 T_0 R_1^2 + 32 R_1^2 R_0 (\sin 4\theta_1 \sin 4\theta_0 + \cos 4\theta_0 \cos 4\theta_1)$$

$$|Q_p| = 16 T_1 T_0^2 - 16 T_1 R_0^2 - 32 T_0 R_1^2 + 32 R_1^2 R_0 \cos 4(\theta_0 - \theta_1) \tag{42}$$

2-Determination of the polar matrix $[S_p]$ as a function of the polar coordinates of [Q]

$[S_p] = [Q_p]^{-1}$

$$S_{p\,11} = (Q_{p\,22}\, Q_{p\,66} - Q_{p\,26}^2) / |Q_p|$$
$$= [4\, T_{0Q}^2 - 4R_{0Q}^2 \cos^2 4\theta_{0Q} - 4\, R_{0Q}^2 \sin^2 4\theta_{0Q}] / |Q_p|$$
$$= [4\, T_{0Q}^2 - 4R_{0Q}^2] / |Q_p|$$

$$S_{p\,12} = (Q_{p\,16}\, Q_{p\,26} - Q_{p\,12}\, Q_{p\,66}) / |Q_p|$$
$$= [8R_{1Q}\, R_{0Q} \sin 2\theta_{1Q} \sin 4\theta_{0Q} - 4\, R_{1Q} \cos 2\theta_{1Q}\, (2T_{0Q} - 2R_{0Q} \cos 4\theta_{0Q})] / |Q_p|$$
$$= [-8\, T_{0Q}\, R_{1Q} \cos 2\theta_{1Q} + 8\, R_{1Q}\, R_{0Q} \cos(4\theta_{0Q} - 2\theta_{1Q})] / |Q_p|$$

$$S_{p\,16} = (Q_{p\,12}\, Q_{p\,26} - Q_{p\,22}\, Q_{p\,16}) / |Q_p|$$
$$= [-8\, T_{0Q}\, R_{1Q} \cos 2\theta_{1Q} + 8\, R_{1Q} R_{0Q}\, (\cos 2\theta_{1Q} \sin 4\theta_{0Q} - \cos 4\theta_{0Q} \sin 2\theta_{1Q})] / |Q_p|$$
$$= [-8\, T_{0Q}\, R_{1Q} \sin 2\theta_{1Q} + 8\, R_{0Q}\, R_{1Q} \sin(4\theta_{0Q} - 2\theta_{1Q})] / |Q_p|$$

$$S_{p\,22} = (Q_{p\,11}\, Q_{p\,66} - Q_{p\,16}^2) / |Q_p|$$
$$= [4\, T_{1Q}\, (2T_{0Q} + 2\, R_{0Q} \cos 4\theta_{0Q}) - 16\, R_{1Q}^2 \sin^2 2\theta_{1Q}] / |Q_p|$$
$$= [\, 8T_{1Q}\, T_{0Q} - 8\, T_{1Q} R_{0Q} \cos 4\theta_{0Q} - 16\, R_{1Q}^2 \sin^2 2\theta_{1Q}] / |Q_p|$$

$$S_{p\,66} = (Q_{p\,11}\, Q_{p\,22} - Q_{p\,12}^2) / |Q_p|$$
$$= [4\, T_{1Q}\, (2\, T_{0Q} + 2R_{0Q} \cos 4\theta_{0Q}) - 16\, R_{1Q}^2 \cos^2 2\theta_{1Q}] / |Q_p|$$
$$= [\, 8T_{1Q}\, T_{0Q} + 8T_{1Q}\, R_{0Q} \cos 4\theta_0 - 16\, R_{1Q}^2 \cos^2 2\theta_{1Q}] / |Q_p|$$

$$S_{p\,26} = (Q_{p\,12}\, Q_{p\,16} - Q_{p\,11}\, Q_{p\,26}) / |Q_p|$$
$$= [16\, R_{1Q}^2 \cos 2\theta_{1Q} \sin 2\theta_{1Q} - 8\, T_{1Q}\, R_{0Q} \sin 4\theta_{0Q}] / |Q_p|$$

3-Components of $[S_p]$ as a Function of the Polar Coordinates of [S]

$$[S_p] = \begin{bmatrix} 4T_{1S} & 4R_{1S}\cos 2\theta_{1S} & 4R_{1S}\sin 2\theta_{1S} \\ 4R_{1S}\cos 2\theta_{1S} & 2T_{0S} + 2R_{0S}\cos 4\theta_{0S} & 2R_{0S}\sin 4\theta_{0S} \\ 4R_{1S}\sin 2\theta_{1S} & 2R_{0S}\sin 4\theta_{0S} & 2T_{0S} - 2R_{0S}\cos 4\theta_{0S} \end{bmatrix}$$

$$S_{p\,11} = 4\,T_{1S} = [4\,T_{0Q}^2 - 4R_{0Q}^2] \,/\, |Q_p|$$

$$S_{p\,12} = 4\,R_{1S}\cos 2\theta_{1S} = [-8\,T_{0Q}\,R_{1Q}\cos 2\theta_{1Q} + 8\,R_{1Q}\,R_{0Q}\cos(4\theta_{0Q} - 2\theta_{1Q})] / |Q_p|$$

$$S_{p\,16} = 4\,R_{1S}\sin 2\theta_{1S} = [-\,8\,T_{0Q}\,R_{1Q}\sin 2\theta_{1Q} + 8\,R_{0Q}\,R_{1Q}\sin(4\theta_{0Q} - 2\theta_{1Q})] \,/\, |Q_p|$$

$$S_{p\,22} = 2\,T_{0S} + 2R_{0S}\cos 4\theta_{0S} = [\,8T_{1Q}\,T_{0Q} - 8\,T_{1Q}\,R_{0Q}\cos 4\theta_{0Q} - 16\,R_{1Q}^2\sin^2 2\theta_{1Q}] \,/\, |Q_p|$$

$$S_{p\,66} = 2\,T_{0S} - 2R_{0S}\cos 4\theta_{0S} = [\,8T_{1Q}\,T_{0Q} + 8T_{1Q}\,R_{0Q}\cos 4\theta_0 - 16\,R_{1Q}^2\cos^2 2\theta_{1Q}] \,/\, |Q_p|$$

$$S_{p\,26} = 2\,R_{0S}\sin 4\theta_{0S} = [16\,R_{1Q}^2\cos 2\theta_{1Q}\sin 2\theta_{1Q} - 8\,T_{1Q}\,R_{0Q}\sin 4\theta_{0Q}] \,/\, |Q_p|$$

4-Polar Co-ordinates of S as a Function of Q Ones.

$$R_{1S}e^{2i\theta 1S} = [2R_{1Q}e^{-2i\theta 1Q}R_{0Q}e^{4i\theta 0Q} - 2T_{0Q}R_{1Q}e^{2i\theta 1Q}] \,/\, |Q_p|$$

$$R_{0S}e^{4i\theta 0S} = [-\,4T_{1Q}R_{0Q}e^{4i\theta 0Q} + 4(R_{1Q}e^{2i\theta 1Q})^2] \,/\, |Q_p| \qquad (43)$$

$$T_{1S} = (T_{0Q}^2 - R_{0Q}^2) \,/\, |Q_p|$$

$$T_{0S} = 4(T_{0Q}T_{1Q} - R_{1Q}^2) \,/\, |Q_p|$$

$$|Q_p| = 16\,T_{1Q}T_{0Q}^2 - 16\,T_{1Q}R_{0Q}^2 - 32\,T_{0Q}R_{1Q}^2 + 32\,R_{1Q}^2R_{0Q}\cos 4(\theta_{0Q} - \theta_{1Q})$$

These equations are valid for all 2-D fourth order tensors.

III-DETERMINATION OF THE POLAR COMPONENTS OF A*

Three samples cut in three different directions, α_0, α_1, α_2 are used. A constant loading is applied to each sample and the strain is measured. The loading is applied in the specimen direction such as $a_\sigma = 0$.

For each sample t_ε, r_ε and a_ε are determined in order to obtain three equations for t_ε and three equations for $r_\varepsilon \cos(2a_\varepsilon)$.

$$t_{\varepsilon 1} = 4[t_\sigma T_{1a*} + r_\sigma R_{1a*} \cos 2(\theta_{1a*} + \alpha_0)]$$

$$t_{\varepsilon 2} = 4[t_\sigma T_{1a*} + r_\sigma R_{1a*} \cos 2(\theta_{1a*} + \alpha_1)]$$

$$t_{\varepsilon 3} = 4[t_\sigma T_{1a*} + r_\sigma R_{1a*} \cos 2(\theta_{1a*} + \alpha_2)] \quad (44)$$

$$r_{\varepsilon 1} \cos(2a_{\varepsilon 1}) = 4 R_{1a*} \cos(2\theta_{1a*} + \alpha_0).\ t_\sigma + 2T_{0a*} r_\sigma + 2R_{0a} r_{\sigma*} \cos(4i\theta_{0a*} + \alpha_0)$$

$$r_{\varepsilon 2} \cos(2a_{\varepsilon 2}) = 4 R_{1a*} \cos(2\theta_{1a*} + \alpha_1).\ t_\sigma + 2T_{0a*} r_\sigma + 2R_{0a} r_{\sigma*} \cos(4i\theta_{0a*} + \alpha_1)$$

$$r_{\varepsilon 3} \cos(2a_{\varepsilon 3}) = 4 R_{1a*} \cos(2\theta_{1a*} + \alpha_2).\ t_\sigma + 2T_{0a*} r_\sigma + 2R_{0a} r_{\sigma*} \cos(4i\theta_{0a*} + \alpha_2)$$

By solving this system (six equations, six unknowns), the polar components of a* are determined.

IV-NUMERICAL EXAMPLES:

In order to test the accuracy of the method, different laminates are used. The same ply is used to build uncoupled materials of different natures. To simulate the tensile test method, the software Laminate+ [8] is used. This software allowed the user to build a material and study its mechanical behaviour. The material may be loaded in any direction and stress and strain across the thickness can be observed. The software calculate also the stiffness and compliance matrices.

1- Nearly Isotropic Laminate

Material :

Ply :

Units (Gpa)

Type	Fiber	Matrix	Ex	Ey	ν	G	T_0	T_1	R_0	R_1	θ_0	θ_1
CFRP	T300	N5208	181	10.3	0.28	7.17	26.8	24.7	19.7	21.4	0	0

Laminate :

9 plies

[0/60/-60/60/-60/0/-60/0/60]

This is an uncoupled laminate.

Using relation (13), the components of A* can be calculated.

T_0	T_1	R_0	R_1	θ_0	θ_1
26.77	24.62	0	0	unknown	unknown

This material is nearly isotropic for R_0 and R_1 are equal to zero.

Using relation (43), the components of a* can be calculated.

T_0	T_1	R_0	R_1	θ_0	θ_1
0.00931967	0.00252781	0	0	unknown	unknown

Loading

σ =80/9 Gpa so t_σ = 40/9Gpa and r_σ = 40/9GPa

Values of t_ε and $r_\varepsilon\cos(2a_\varepsilon)$ for θ equal to 0, $\pi/6$, $2\pi/6$ computed with Laminate+ [8]

	0	$\pi/6$	$2\pi/6$
t_ε	0.0445	0.0445	0.0445
$r_\varepsilon\cos(2a_{\varepsilon xfs})$	0.0825	0.0825	0.0825

These values can be used as results of practical tests in order to check the validity of the method.Using these values and the system (44) :

$0.0445 = 4[40\ T_{1a*} + 40\ R_{1a*}\cos 2(\theta_{1a*} + 0)]/9$

$0.0445 = 4[40\ T_{1a*} + 40\ R_{1a*}\cos 2(\theta_{1a*} + \pi/6)]/9$

$0.0445 = 4[40\ T_{1a*} + 40\ R_{1a*}\cos 2(\theta_{1a*} + 2\pi/6)]/9$

$0.0825 = 4\ R_{1a*}\cos(2\theta_{1a*} + 0).\ 40/9 + 2T_{0a*}\ 40/9 + 2R_{0a*}\ 40/9\cos(4i\theta_{0a*} + 0)$

$0.0825 = 4\ R_{1a*}\cos(2\theta_{1a*} + \pi/6).\ 40/9 + 2T_{0a*}\ 40/9 + 2R_{0a*}\ 40/9\cos(4i\theta_{0a*} + \pi/6)$

$0.0825 = 4\ R_{1a*}\cos(2\theta_{1a*} + 2\pi/6).\ 40/9 + 2T_{0a*}\ 40/9 + 2R_{0a*}\ 40/9\cos(4i\theta_{0a*} + 2\pi/6)$

By solving the system (44), the following values are obtained :

T'_0	T'_1	R'_0	R'_1	θ'_0	θ'_1
0.00931967	0.00928125	0	0	unknown	unknown

Errors :

$(T_0 - T'_0)/T_0$	$(T1 - T'_1)/T_1$	$(R_0 - R'_0)/R_0$	$(R_1 - R'_1)/R_1$	$(\theta_0 - \theta'_0)/\theta_0$	$(\theta_1 - \theta'_1)/\theta_1$
0.00095	0.00176	-0.00651	0.00590	0	0

2-Other Material

Material :

Ply :

Units (Gpa)

Type	Fiber	Matrix	Ex	Ey	ν	G	T_0	T_1	R_0	R_1	θ_0	θ_1
CFRP	T300	N5208	181	10.3	0.28	7.17	26.8	24.7	19.7	21.4	0	0

Laminate :

7 plies

[0/45/45/90/0/0/45]

This is an uncoupled laminate.

Using relation (13), the components of A* can be calculated.

T_0	T_1	R_0	R_1	θ_0	θ_1
26.77	24.62	2.77	10.96	0	0.9865

Using relation (43), the components of a* can be calculated.

T_0	T_1	R_0	R_1	θ_0	θ_1
0.0125245	0.0041197	0.0037169	0.0035656	2.377	4.2195

Loading

σ =80/7 Gpa so t_σ = 40/7 Gpa and r_σ = 40/7 GPa

Values of t_ε and $r_\varepsilon\cos(2a_\varepsilon)$ for θ equal to 0, $\pi/6$, $2\pi/6$ computed with Laminate+[8]

angle	0	$\pi/6$	$2\pi/6$
t_ε	0.055	0.136	0.175
$r_\varepsilon\cos(2a_\varepsilon)$	0.073	0.175	0.265

These values can be used as results of practical tests in order to check the validity of the method.

Using these values and the system (44) :

$0.055 = 4[40\ T_{1a*} + 40\ R_{1a*}\cos2(\theta_{1a*} + 0)]/7$

$0.136 = 4[40\ T_{1a*} + 40\ R_{1a*}\cos2(\theta_{1a*} + \pi/6)]/7$

$0.175 = 4[40\ T_{1a*} + 40\ R_{1a*}\cos2(\theta_{1a*} + 2\pi/6)]/7$

$0.073 = 4\ R_{1a*}\cos(2\theta_{1a*} + 0).\ 40/7 + 2T_{0a*}\ 40/7 + 2R_{0a*}\ 40/7\cos(4i\theta_{0a*} + 0)$

$0.175 = 4\ R_{1a*}\cos(2\theta_{1a*} + \pi/6).\ 40/7 + 2T_{0a*}\ 40/7 + 2R_{0a*}\ 40/7\cos(4i\theta_{0a*} + \pi/6)$

$0.265 = 4\ R_{1a*}\cos(2\theta_{1a*} + 2\pi/6).\ 40/7 + 2T_{0a*}\ 40/7 + 2R_{0a*}\ 40/7\cos(4i\theta_{0a*} + 2\pi/6)$

By solving the system, the following values are obtained :

T'_0	T'_1	R'_0	R'_1	θ'_0	θ'_1
0.0125125	0.0041125	0.00374107	0.00354456	2.38196	4.21017

Errors :

$(T_0 - T'_0)/T_0$	$(T1 - T'_1)/T_1$	$(R_0 - R'_0)/R_0$	$(R_1 - R'_1)/R_1$	$(\theta_0 - \theta'_0)/\theta_0$	$(\theta_1 - \theta'_1)/\theta_1$
0.00095	0.00176	-0.00651	0.00590	0	0

3-Quasi Homogeneous Orthotropic Laminate :

Material :

Ply :

Units (Gpa)

Type	Fiber	Matrix	Ex	Ey	ν	G	T_0	T_1	R_0	R_1	θ_0	θ_1
CFRP	T300	N5208	181	10.3	0.28	7.17	26.8	24.7	19.7	21.4	0	0

Laminate :

8 plies

[30/60/60/30/60/30/30/60]

This is an uncoupled laminate.

Using relation (13), the components of A* can be calculated.

T_0	T_1	R_0	R_1	θ_0	θ_1
26.77	24.62	9.85	17.06	π	$\pi/2$

Using relation (43), the components of a* can be calculated.

T_0	T_1	R_0	R_1	θ_0	θ_1
0.0214	0.01046	0.0067	0.0106	π	$-\pi/2$

Loading

σ =80/8 Gpa so t_σ = 40/8 Gpa and r_σ = 40/8 GPa

Values of t_ε and $r_\varepsilon\cos(2a_\varepsilon)$ for θ equal to 0, π/6, 2π/6 computed with Laminate+[8]

angle	0	π/6	2π/6
t_ε	0.208	0.391	0.391
$r_\varepsilon\cos(2a_\varepsilon)$	0.146	0.43	0.43

These values can be used as results of practical tests in order to check the validity of the method.

Using these values and the system (44) :

$0.208 = 4[40\ T_{1a*} + 40\ R_{1a*}\cos 2(\theta_{1a*} + 0)]/8$

$0.391 = 4[40\ T_{1a*} + 40\ R_{1a*}\cos 2(\theta_{1a*} + \pi/6)]/8$

$0.391 = 4[40\ T_{1a*} + 40\ R_{1a*}\cos 2(\theta_{1a*}+ 2\pi/6)]/8$

$0.146 = 4\ R_{1a*}\cos(2\theta_{1a*}+ 0).\ 40/8+ 2T_{0a*}\ 40/8+ 2R_{0a*}\ 40/8\cos(4i\theta_{0a*}+ 0)$

$0.43 = 4\ R_{1a*}\cos(2\theta_{1a*}+ \pi/6).\ 40/8+ 2T_{0a*}\ 40/7 + 2R_{0a*}\ 40/8\cos(4i\theta_{0a*}+ \pi/6)$

$0.43= 4\ R_{1a*}\cos(2\theta_{1a*}+ 2\pi/6).\ 40/8 + 2T_{0a*}\ 40/7+ 2R_{0a*}\ 40/8\cos(4i\theta_{0a*}+ 2\pi/6)$

By solving the system, the following values are obtained :

T'_0	T'_1	R'_0	R'_1	θ'_0	θ'_1
0.0213	0.01039	0.0067	0.0106	π	-π/2

Errors :

$(T_0 - T'_0)/T_0$	$(T1 - T'_1)/T_1$	$(R_0 - R'_0)/R_0$	$(R_1 - R'_1)/R_1$	$(\theta_0 - \theta'_0)/\theta_0$	$(\theta_1 - \theta'_1)/\theta_1$
0.0046	0.0066	0	0	0	0

4-Square Symmetric, Quasi Homogeneous Laminate :

Material :

Ply :

Units (Gpa)

Type	Fiber	Matrix	Ex	Ey	ν	G	T_0	T_1	R_0	R_1	θ_0	θ_1
CFRP	T300	N5208	181	10.3	0.28	7.17	26.8	24.7	19.7	21.4	0	0

Laminate :

8 plies

[18/-72/-72/18/-72/18/18/-72]

This is an uncoupled laminate.

Using relation (13), the components of A* can be calculated.

T_0	T_1	R_0	R_1	θ_0	θ_1
26.77	24.62	19.69	0	1.25	unknown

Using relation (43), the components of a* can be calculated.

T_0	T_1	R_0	R_1	θ_0	θ_1
0.0203	0.00253	0.0149	0	1.099	unknown

Loading

σ =80/8 Gpa so t_σ = 40/8 Gpa and r_σ = 40/8 GPa

Values of t_ε and $r_\varepsilon\cos(2a_\varepsilon)$ for θ equal to 0, π/6, 2π/6 computed with Laminate+[8]

angle	0	π/6	2π/6
t_ε	0.05	0.05	0.05
$r_\varepsilon\cos(2a_\varepsilon)$	0.155	0.345	0.102

These values can be used as results of practical tests in order to check the validity of the method.

Using these values and the system (44) :

$0.05 = 4[40\ T_{1a*} + 40\ R_{1a*}\cos2(\theta_{1a*} + 0)]/8$

$0.05 = 4[40\ T_{1a*} + 40\ R_{1a*}\cos2(\theta_{1a*} + \pi/6)]/8$

$0.05 = 4[40\ T_{1a*} + 40\ R_{1a*}\cos2(\theta_{1a*} + 2\pi/6)]/8$

$0.155 = 4\ R_{1a*}\cos(2\theta_{1a*} + 0).\ 40/8 + 2T_{0a*}\ 40/8 + 2R_{0a*}\ 40/8\cos(4i\theta_{0a*} + 0)$

$0.345 = 4\ R_{1a*}\cos(2\theta_{1a*} + \pi/6).\ 40/8 + 2T_{0a*}\ 40/7 + 2R_{0a*}\ 40/8\cos(4i\theta_{0a*} + \pi/6)$

$0.102 = 4\ R_{1a*}\cos(20_{1a*} + 2\pi/6).\ 40/8 + 2T_{0a*}\ 40/7 + 2R_{0a*}\ 40/8\cos(4i\theta_{0a*} + 2\pi/6)$

By solving the system, the following values are obtained :

T'_0	T'_1	R'_0	R'_1	θ'_0	θ'_1
0.0201	0.0025	0.0147	0	1.097	unknown

Errors :

$(T_0 - T'_0)/T_0$	$(T1 - T'_1)/T_1$	$(R_0 - R'_0)/R_0$	$(R_1 - R'_1)/R_1$	$(\theta_0 - \theta'_0)/\theta_0$	$(\theta_1 - \theta'_1)/\theta_1$
0.0099	0.012	0.013	0	0	0

D-SOFTWARE PRESENTATION

I-INTRODUCTION TO THE SOFTWARE

The idea is to provide a program able to determine the compliance and stiffness matrices of an uncoupled composite material.

A tensile test is applied to three samples cut in three different directions.

If strain gages are used the user does not need any knowledge in mechanical behaviour. He/she is expected to test the material and to enter the measured strains in the program.

The user may use any of the other measurements described in part A but in this case he musts be able to calculate the strain state for each sample.

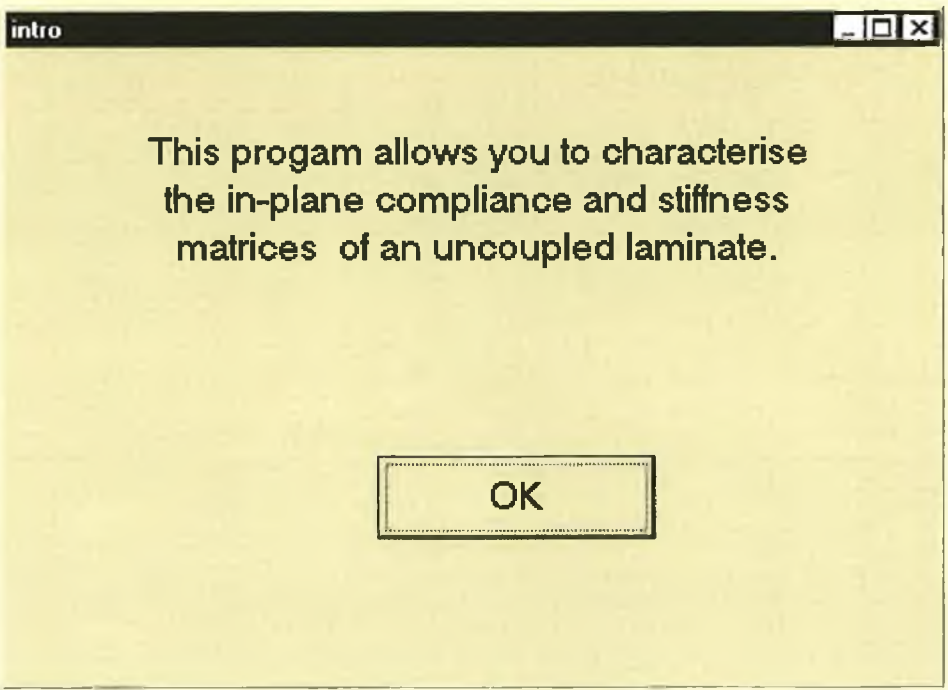
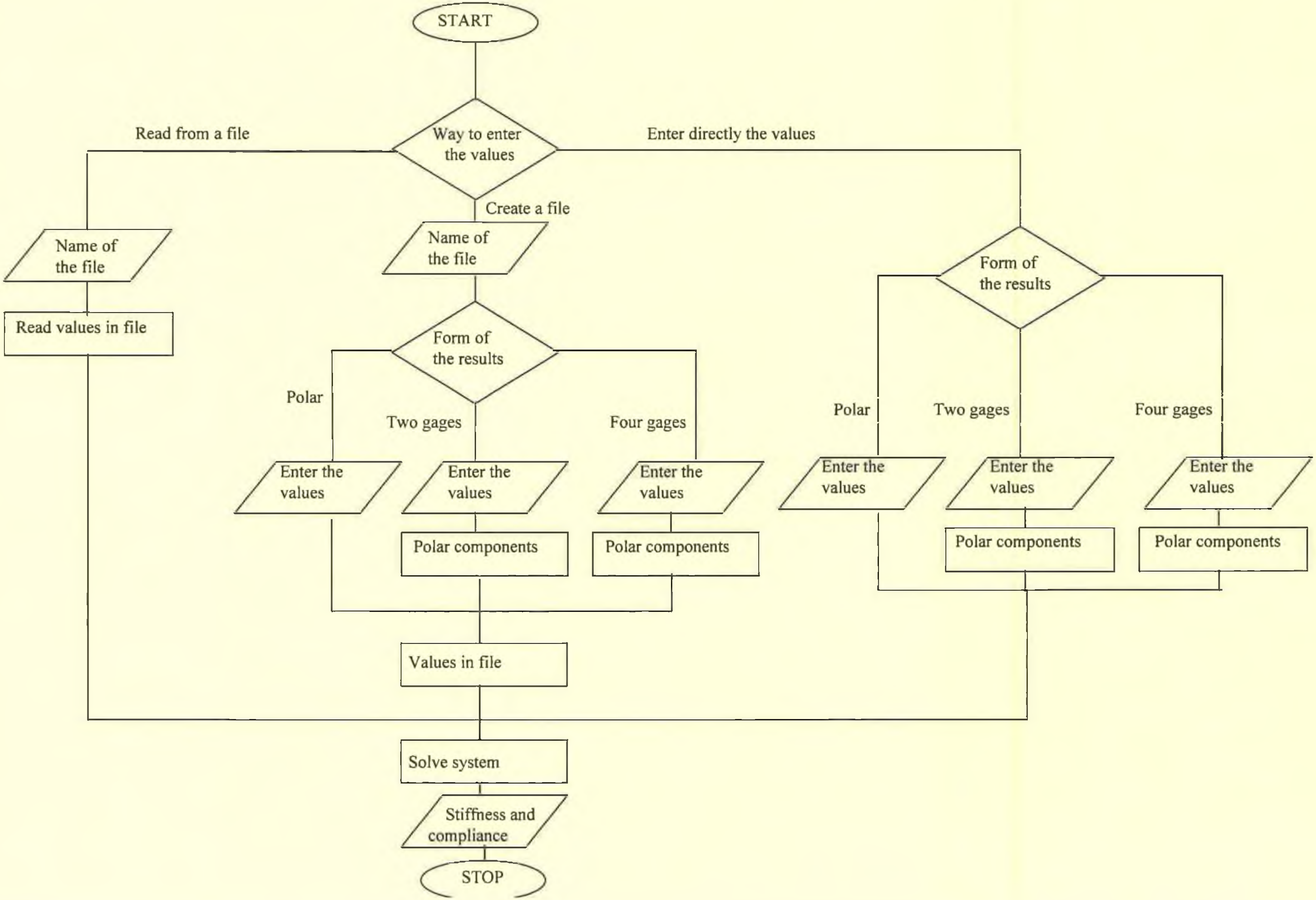


Fig. 18 Software introduction page

II FLOW CHART OF THE PROGRAM

Next page shows the program organisation.


START
Way to enter the values
Read from a file
Create a file
Enter directly the values
Name of the file
Read values in file
Name of the file
Form of the results
Polar
Two gages
Four gages
Enter the values
Enter the values
Enter the values
Polar components
Polar components
Values in file
Form of the results
Polar
Two gages
Four gages
Enter the values
Enter the values
Enter the values
Polar components
Polar components
Solve system
Stiffness and compliance
STOP

III-DATA

1-Data Input Choices

The first menu presents ways to input data. The user can create a file which can be useful to compare with other tests later on. He can use a file previously created. Or, the third option is to enter directly the values, knowing that the computer will not keep any trace.

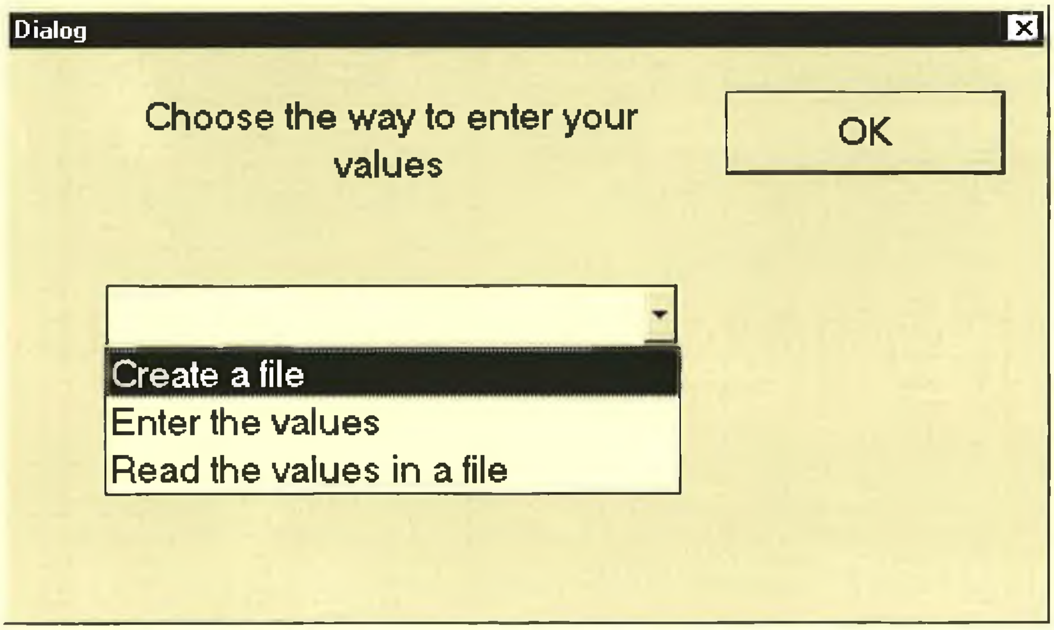


Fig.19 Way to enter the values

2-Choice of Measurement

The user musts tell the software which kind of measurement he has done.

There are three possibilities :

First, using strain gages, the easiest way is to use two gages out of phase by $\pi/2$ for each sample. Indeed, the third gage is not necessary because r sin2a is not useful to solving the system (44).

Second, four gages can be used as explained in chapter B. The program uses the least squares method to find the polar co-ordinates of the strain.

Third, any kind of measurements can be use but the user musts calculate the polar co-ordinates by himself/herself.

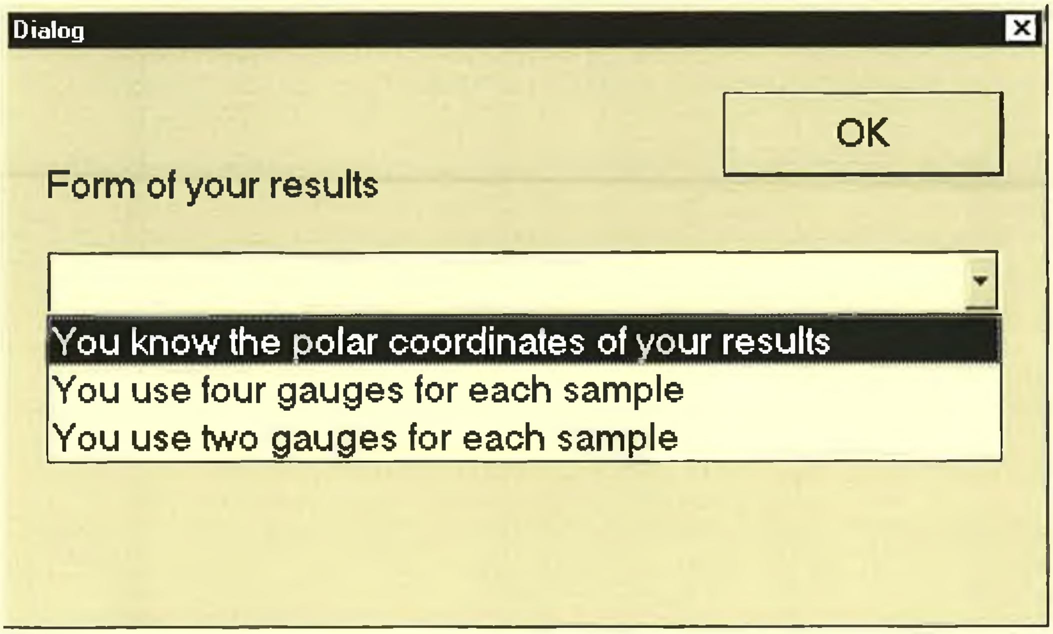


Fig.20 Choice of measurement

3-Stress and Angles in which the Samples are Cut

Then, the program needs the stress applied in Gpa. The code calculates itself the polar coordinates of the stress.

The directions in which the user cut his three samples is not imposed. However, one of the sample is chosen as a reference and the angles of the two others are defined as the difference between the direction in which they are cut and the direction of reference.

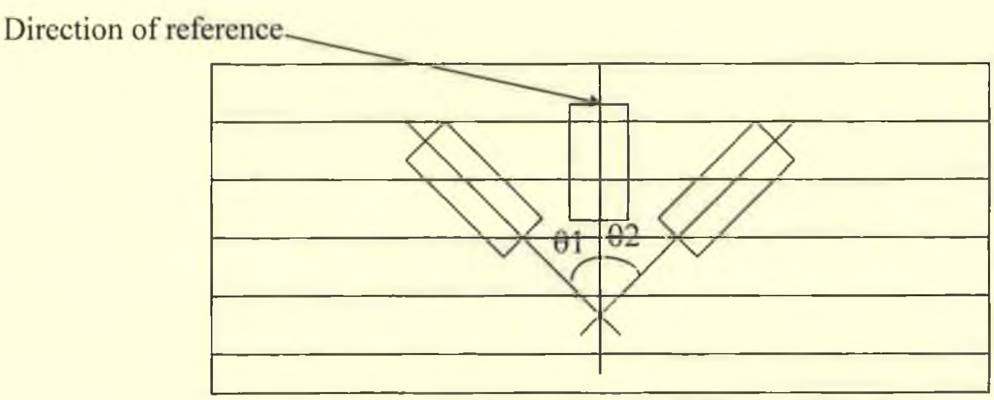


Fig. 21 Directions in which the samples are cut

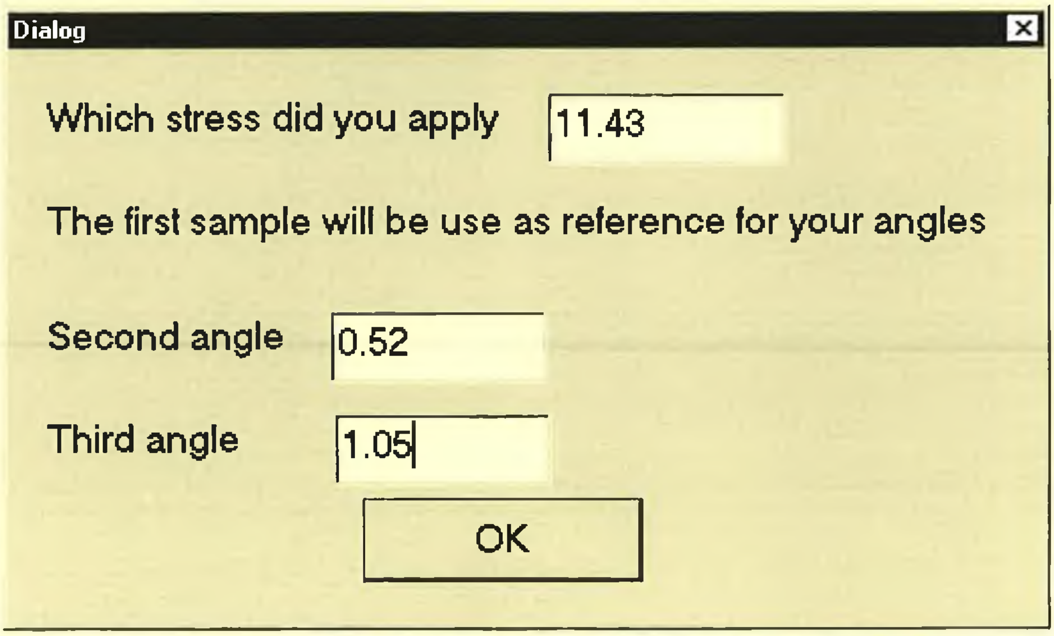


Fig.22 Stress and angle in which the samples are cut

4-Tests Results

For each sample the user enters the test results. The presentation depends on the type of measurement the user has chosen in part III.2. Fig.23 shows the dialogue boxes corresponding to four gages for each sample.

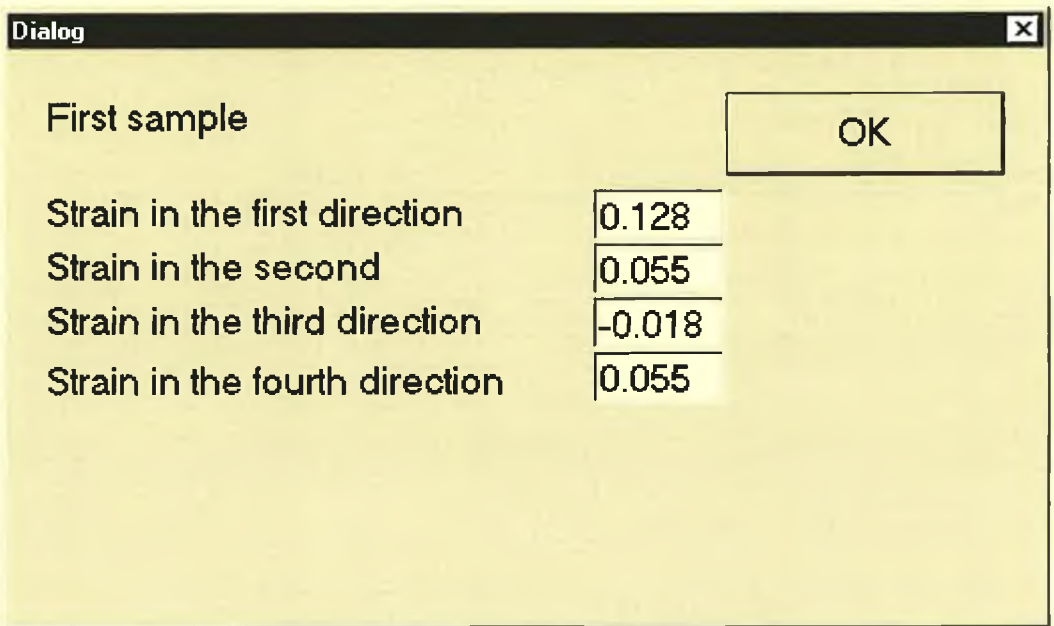


Fig.23 Results of tests for the third sample

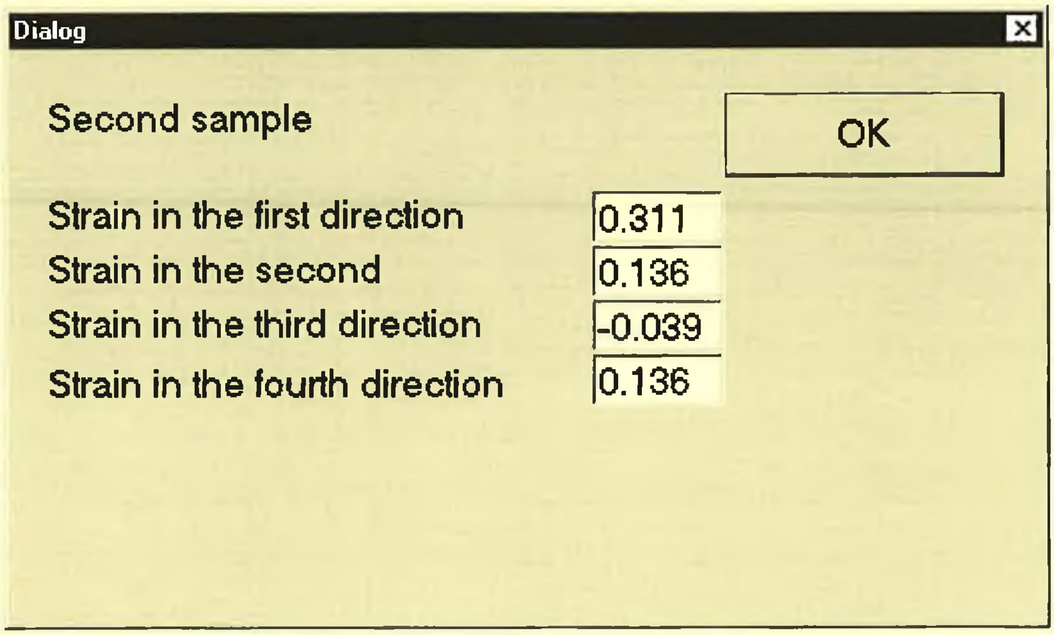


Fig.24 Results of tests for the second sample

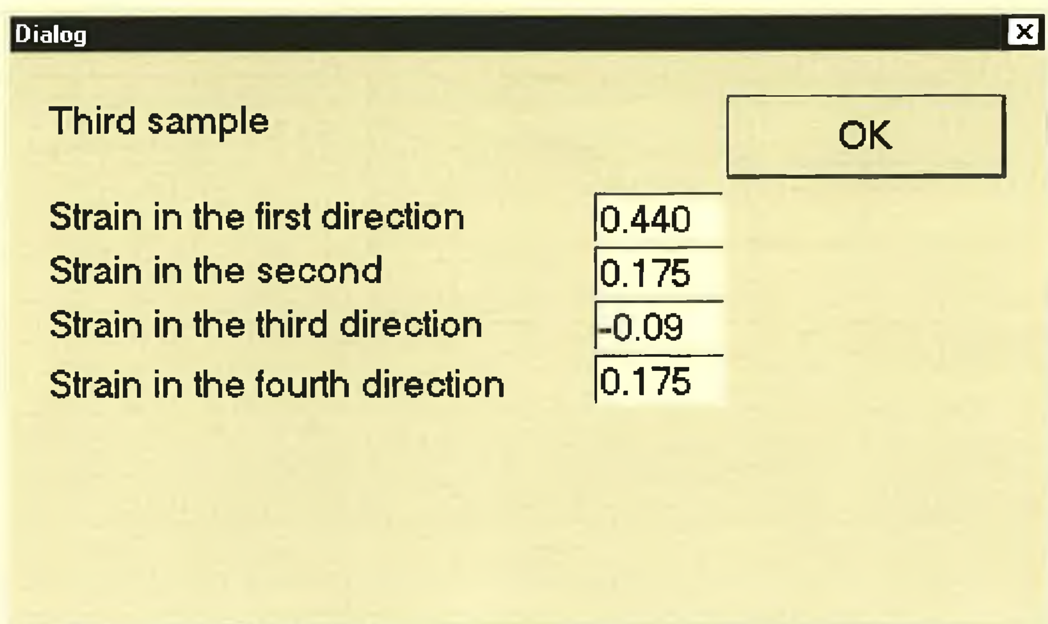


Fig.25 Results of tests for the third sample

IV-CALCULATION AND RESULTS PRESENTATION

The last step is the results. They are presented first in polar and then in cartesian coordinates.

1-Polar Components of the Compliance Matrix

The system (44) is used to determine the polar components of the compliance matrix.

$$t_{\varepsilon 1} = 4[t_\sigma\ T_{1a*} + r_\sigma\ R_{1a*} \cos 2(\theta_{1a*} + \alpha_0)]$$

$$t_{\varepsilon 2} = 4[t_\sigma\ T_{1a*} + r_\sigma\ R_{1a*} \cos 2(\theta_{1a*} + \alpha_1)]$$

$$t_{\varepsilon 3} = 4[t_\sigma\ T_{1a*} + r_\sigma\ R_{1a*} \cos 2(\theta_{1a*} + \alpha_2)]$$

$$r_{\varepsilon 1} \cos(2a_{\varepsilon 1}) = 4\ R_{1a*} \cos(2\theta_{1a*} + \alpha_0).\ t_\sigma + 2T_{0a*}\ r_\sigma + 2R_{0a}\ r_{\sigma*} \cos(4i\theta_{0a*} + \alpha_0)$$

$$r_{\varepsilon 2} \cos(2a_{\varepsilon 2}) = 4\ R_{1a*} \cos(2\theta_{1a*} + \alpha_1).\ t_\sigma + 2T_{0a*}\ r_\sigma + 2R_{0a}\ r_{\sigma*} \cos(4i\theta_{0a*} + \alpha_1)$$

$$r_{\varepsilon 3} \cos(2a_{\varepsilon 3}) = 4\ R_{1a*} \cos(2\theta_{1a*} + \alpha_2).\ t_\sigma + 2T_{0a*}\ r_\sigma + 2R_{0a}\ r_{\sigma*} \cos(4i\theta_{0a*} + \alpha_2)$$

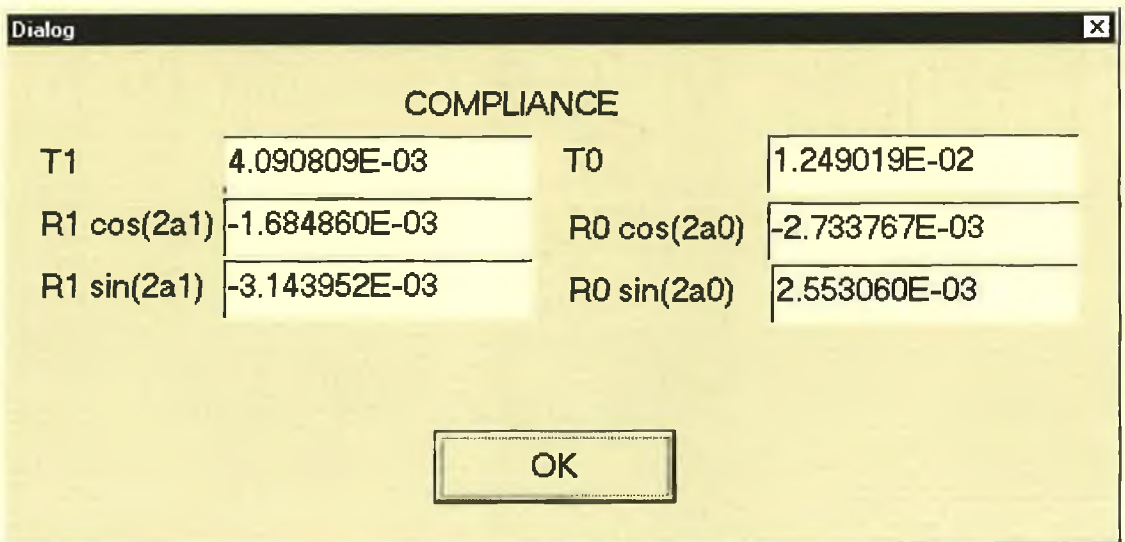


Fig.26 Polar components of the compliance matrix

2-Cartesian Coordinates of the Compliance Matrix

The relations (12) give the cartesian coordinates of the compliance matrix.

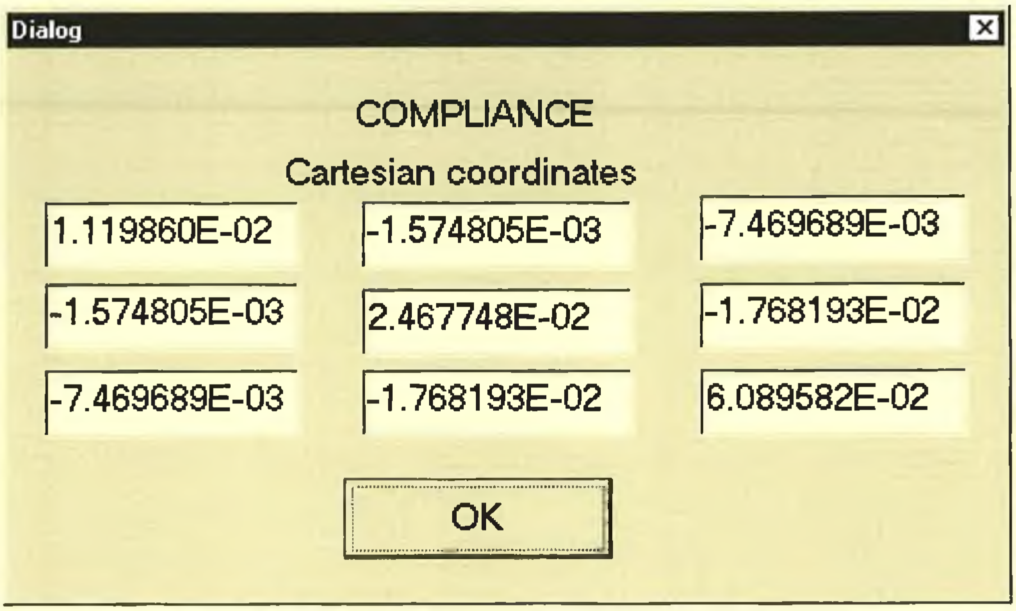


Fig.27 Cartesian coordinates of the compliance matrix

3-Polar Components of the Stiffness Matrix

The relations (43) enable the calculation of the polar components of the stiffness matrix.

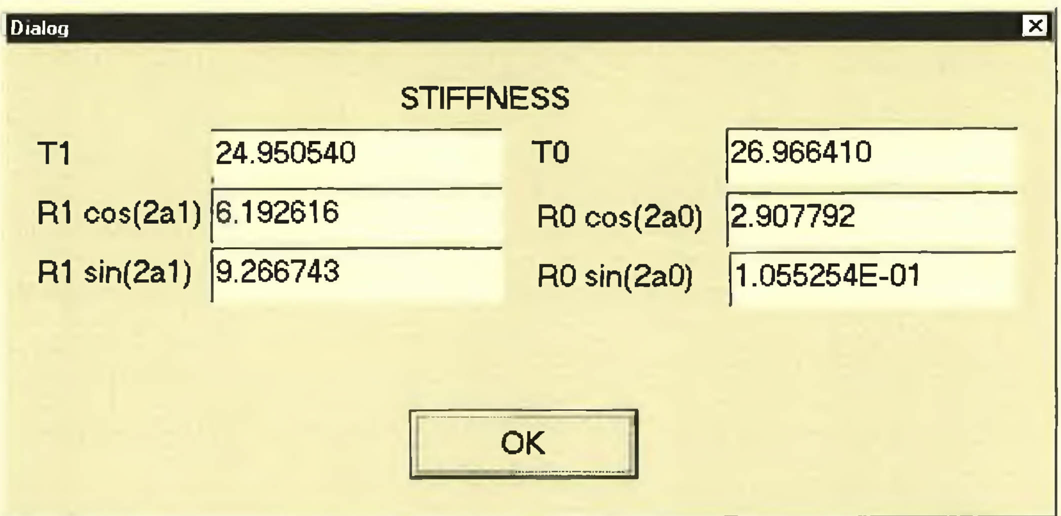


Fig.28 Polar coordinates of the compliance matrix

4- Cartesian Coordinates of the Stiffness Matrix

The relations (12) gives the cartesian coordinates of the stiffness matrix.

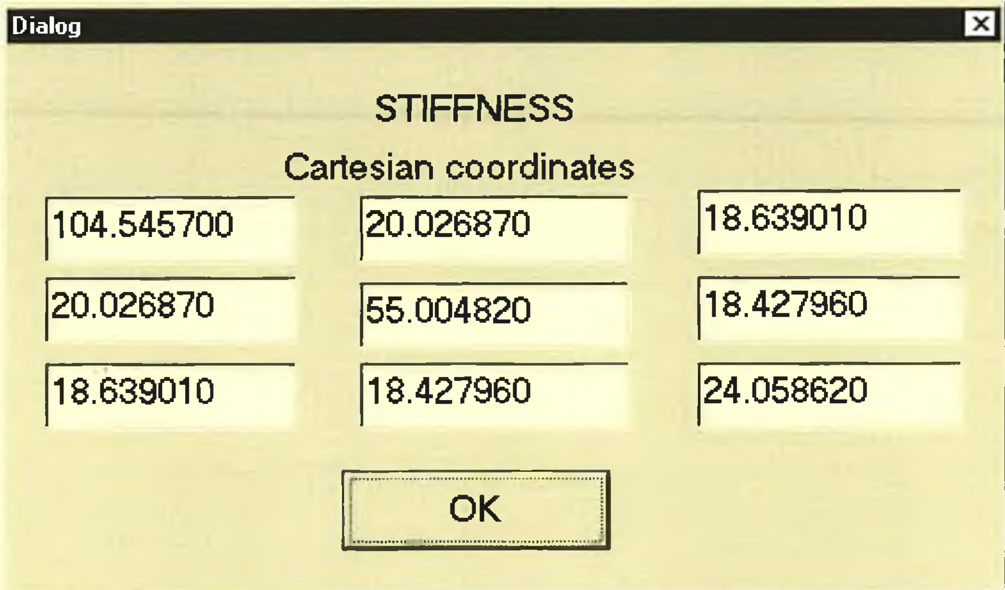


Fig.29 Cartesian coordinates of the compliance matrix

5-Printing

The results may be printed at the end of the application.

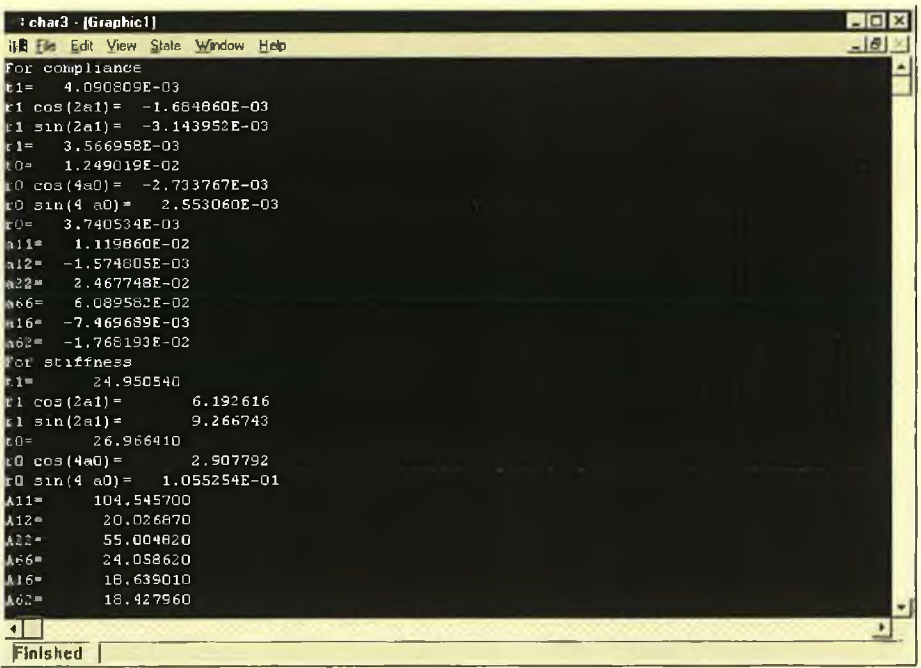


Fig.30 Results printing

V- CODES

The codes can be found in appendix C.

VI- FURTHER APPLICATIONS.

1-Calculations

The program can be used to calculate the polar components of the different matrices versus their cartesian components and vice-versa.

It can be also used to calculate the stiffness matrices versus the compliance matrices and vice-versa.

2-Materials Study

Using the properties of “quasi” and “nearly” described in section A-5, the program could be developed in order to determine the materials properties and then simplify the study. For example, if a material is “quasi” orthotropic, it could be studied as an orthotropic material.

CONCLUSION

The aim of this work was to develop a method for the characterisation of in-plane compliance and stiffness matrices of uncoupled composite materials.

The first part of the study concerned the experimentation. It seemed interesting to study the strain-gages because they are the easiest and cheapest measurement tools.

In order to minimise the influence of an experimentation error on the strain state calculated, the advantages of using four gages instead of three have been studied. The Least Squares Method has been chosen to calculate strain state (using four gages the system became over-determined). The influence of an experimentation error on the final result depends on the directions in which the gages are bound to the material. The best results are obtained when binding the gages at 0°, 45°, 90° and 135°(if 0° is the material principal direction). Finally, an error calculation has been done, allowing the user to correct the measurements if some errors are identified.

The second part concerns the characterisation of the elasticity coefficients. The decision has been taken to use the polar method introduced by Verchery and Vong [2]. The main advantage of this method is a direct access to any direction of the material. Firstly, the strain-stress relation and the compliance-stiffness relation have been developed using the polar components. Then a system of six equations has been presented. Using a tensile test method, a constant stress is applied to three samples cut in three different directions of the material and the strain state is calculated for each sample. Introducing the strain and stress states in the system of six equations, the solutions are the polar components of the compliance matrix of the material. Four numerical examples are given showing the good accuracy of the method.

In the last part, a Fortran program is presented as a tool for the experimentation. The program builds the system of six equations using the data given by the user. Then, it calculates the solutions. Finally, using the relations developed in the second part, it outputs the stiffness and compliance matrices in polar and cartesian coordinates.

Further improvements

Further improvements to the Fortran program could be done in order to help the user to analyse the material properties.

This work represents a progress in the study of mechanical behaviour of composite materials. But it concerns only the in-plane behaviour of uncoupled laminates (however, the method can be used for any uncoupled material).

The system of six equations would be valid for the flexure, but the experimental approach would not .

Then, for the coupled material the strain-stress relation is different and much more complex so the present work can be used as a basis for the study but is not valid anymore.

I believe that my research will, in some part, contribute to the development in understanding of the behaviour of composite materials for future researchers involved in this area.

APPENDIX A

RANDOM ERRORS

In order to test the accuracy of the gages samples, a Mohr circle of given radius (r=12) and centre (t=24) is built. With this tool, the strain corresponding to each of the angles chosen for the gages, can be determined and these values can be used as measured values.

The aim is to attribute a random error to each of these values. For this, we assume that the errors are normally distributed with mean 0 and variance 1. We discretise the repartition curve in order to obtain the probabilities for -4, -3, -2, -1, 0, 1, 2, 3, 4.

Error	Probability	%	Frequency chosen
-4	0.0002	0.02	1
-3	0.006	0.6	2
-2	0.0606	6.06	4
-1	0.2417	24.17	24
0	0.383	38.3	38
1	0.2417	24.17	24
2	0.0606	6.06	4
3	0.006	0.6	2
4	0.0002	0.02	1

If we use two digits to number each of the 100 possibilities, we can assign 0 to -4, 1 and 2 to -3, etc…

Error	Frequency chosen	Sampling number
-4	1	0
-3	2	1 - 2
-2	4	3 - 6
-1	24	7 - 30
0	38	31 - 68
1	24	69 - 92
2	4	93 - 96
3	2	97 - 98
4	1	99

Hence, as many samples of four values as we want can be created by simulating a draw of numbers between 0 and 99. For this, the random function of a package such as Mathematica or a random table number can be used.

The test can be done for samples of three or four gages at 0, $\pi/4$, $\pi/2$ and $3\pi/4$ for the fourth gage.

Samples obtained, corresponding errors and the Mohr circle radius and centre calculated for three and four gages :

Sample number drawn	Errors	r 4 gages	t 4 gages	r 3 gages	t 3 gages
51-77-27-46	0 1 -1 0	12.5	24	12.6	23.5
40-42-33-12	0 0 0 -1	12	23.8	12	24
90-44-46-62	2 0 0 0	13	24.5	13	25
12-40-33-23	-1 0 0 -1	11.5	23.5	11.5	23.5
49-18-35-87	0 -1 0 1	12	24	12	24
06-56-82-19	-2 0 1 -1	10.5	23.5	10.5	23.5
60-45-93-96	0 0 2 2	11	25	11	25
01-75-52-07	-3 1 0 -1	10.5	23.3	10.8	22.5
82-54-24-11	1 0 -1 -1	13	23.8	13	24
64-53-05-86	0 0 -2 1	13	23.8	13	23
02-13-37-57	-3 -1 0 0	10.5	23	10.5	22.5
97-45-40-63	3 0 0 0	13.5	24.8	13.6	25.5
10-41-03-58	-1 0 -2 0	12.5	23.3	12.6	22.5
57-93-53-81	0 2 0 1	12	24.8	12.2	24
93-88-23-22	2 1 -1 -1	13.5	24.3	13.5	24.5
96-79-96-49	2 1 2 0	12	25.3	12	26
37-03-35-59	0 -2 0 0	12	23.5	12.2	24
58-63-20-79	0 0 -1 1	12.5	24	12.5	23.5
06-53-04-15	-2 0 -2 -1	12	22.8	12.2	22
63-06-47-59	0 -2 0 0	12	23.5	12.2	24
51-13-59-85	0 -1 0 1	12	24	12	24
27-62-58-60	-1 0 0 0	11.5	23.8	11.5	23.5
94-48-56-30	2 0 0 -1	13	24.3	13	25
15-76-83-30	-1 1 1 -1	11	24	11	24
27-79-46-23	-1 1 0 -1	11.5	23.8	11.6	23.5
23-21-63-19	-1 -1 0 -1	11.5	23.3	11.5	23.5
11-57-77-33	-1 0 1 0	11	24	11	24
16-07-10-52	-1 -1 -1 0	12	23.3	12	23
29-09-10-97	-1 -1 -1 3	12.2	24	12	23
17-48-02-94	-1 0 -3 2	13	23.5	13.2	22
14-06-82-98	-1 -2 1 3	11.3	24.3	11.2	24
78-43-50-53	1 0 0 0	12.5	24.3	12.5	24.5
22-54-96-95	-1 0 2 2	10.5	24.8	10.5	24.5
65-24-24-57	0 -1 -1 0	12.5	23.5	12.5	23.5
73-54-70-71	1 0 1 1	12	24.8	12	25

36-40-87-67	0 0 1 0	11.5	24.3	11.5	24.5
99-71-54-19	4 1 0 -1	14	25	14	26
52-57-08-27	0 0 -1 -1	12.5	23.5	12.5	23.5
98-96-47-28	3 2 0 -1	13.6	25	13.5	25.5
10-74-40-83	-1 1 0 1	11.5	24.3	11.6	23.5
96-56-24-28	2 0 -1 -1	13.5	24	13.5	24.5
51-84-73-94	0 1 1 2	11.5	25	11.5	24.5
93-66-01-46	2 0 -3 0	14.5	23.8	14.5	23.5
25-30-04-77	-1 -1 -2 0	12.5	23	12.5	22.5
54-39-82-11	0 0 1 -1	11.5	24	11.5	24.5
54-97-81-03	0 3 1 -2	11.8	24.5	11.8	24.5
67-64-32-86	0 0 0 1	12	24.3	12	24
96-55-44-34	2 0 0 0	13	24.5	13	25
37-10-95-91	0 -1 2 1	11	24.5	11.2	25
07-83-95-88	-1 1 2 1	10.5	24.8	10.5	24.5
92-08-26-36	1 -1 -1 0	13	23.8	13	24
59-52-85-76	0 0 1 1	11.5	24.5	11.5	24.5
26-42-36-39	-1 0 0 0	11.5	23.8	11.5	23.5
23-83-98-17	-1 1 3 -1	10	24.5	10	25
67-90-64-82	0 1 0 1	12	24.5	12	24
36-16-05-78	0 -1 -2 1	13	23.5	13	23
18-12-62-25	-1 -1 0 -1	11.5	23.3	11.5	23.5
70-40-77-79	1 0 1 1	12	24.8	12	25
44-39-20-30	0 0 -1 -1	12.5	23.5	12.5	23.5
24-79-53-29	-1 1 0 -1	11.5	23.8	11.6	23.5
89-02-76-62	1 -3 1 0	12.1	23.8	12	25
54-52-19-44	0 0 -1 0	12.5	23.8	12.5	23.5
16-53-06-28	-1 0 -2 -1	12.5	23	12.6	22.5
98-93-58-20	3 2 0 -1	13.6	25	13.5	25.5
41-86-19-64	0 1 -1 0	12.5	24	12.5	24
08-70-56-97	-1 1 0 3	11.5	24.8	11.6	23.5
43-74-28-93	0 1 -1 2	12.5	24.5	12.5	24
50-79-42-71	0 1 0 1	12	24.5	12	24
86-70-71-29	1 1 1 -1	12	24.5	12	25
73-17-16-98	1 -1 -1 3	13.1	24.5	13	24
81-16-42-18	1 -1 0 -1	12.5	23.8	12.6	24.5
78-56-59-36	1 0 0 0	12.5	24.3	12.5	24.5
08-15-08-84	-1 -1 -1 1	12	23.5	12	23
14-07-07-49	-1 -1 -1 0	12	23.3	12	23
50-55-18-90	0 0 -1 1	12.5	24	12.5	23.5
07-45-65-25	-1 0 0 -1	11.5	23.5	11.5	23.5
31-18-22-15	0 -1 -1 -1	12.5	23.3	12.5	23.5
80-44-18-89	1 0 -1 1	13	24.3	13	24
25-43-98-80	-1 0 3 1	10	24.8	10	25
36-24-03-46	0 -1 -2 0	13	23.3	13	23
72-83-85-41	1 1 1 0	12	24.8	12	25

86-88-29-06	1 1 -1 -2	13.1	23.8	13	24
65-24-19-82	0 -1 -1 1	12.5	23.8	12.5	23.5
49-15-91-68	0 -1 1 0	11.5	24	11.6	24.5
35-48-65-37	0 0 0 0	12	24	12	24
15-90-16-15	-1 1 -1 -1	12	23.5	12.2	23
91-46-76-28	1 0 1 -1	12	24.3	12	25
19-52-72-79	-1 0 1 1	11	24.3	11	24
47-15-23-56	0 -1 -1 0	12.5	23.5	12.5	23.5
83-47-28-07	1 0 -1 -1	13	23.8	13	24
65-52-50-02	0 0 0 -3	12.1	23.3	12	24
02-48-19-52	-3 0 -1 0	11	23	11.2	22
98-47-61-68	3 0 0 0	13.5	24.8	11.6	25.5
32-72-48-49	0 1 0 0	12	24.3	12	24
49-23-75-93	0 -1 1 2	11.6	24.5	11.6	24.5
51-81-73-67	0 1 1 -2	11.6	24	11.5	24.5
38-77-61-05	0 1 0 -2	12.1	23.8	12	24
10-86-39-74	-1 1 0 1	11.5	24.3	11.6	23.5
53-90-58-15	0 1 0 -1	12	24	12	24
58-70-56-97	0 1 0 3	12	25	12	24

We note the following percentages in favour of each case.

Best approximation	Three gages	Four gages	Same approximation
Value of r	17%	16%	67%
Value of t	27%	54%	19%

Thus, the table shows that the fourth gage gives a better accuracy more often than the three gages.

APPENDIX B

COMPARISON BETWEEN THE DIFFERENT ANGLES IN WHICH THE GAGES CAN BE USED

All the combinations of 10% error are applied to the gages. A cross, in the angle box corresponds to this error. For example, the first test is done with a 10% percent error only in the 0 direction.

Serie 1 2

0	2π/3	4π/3	π/4	r *	r	t *	t	% r*	% r	% t *	% t	|% r*|	|% r|	|%t *|	|%t|
x				11.8	14.4	24.7	25.0	-1.3	20.0	2.8	4.2	1.3	20.0	2.8	4.2
	x			11.3	11.4	24.5	24.7	-5.6	-4.8	2.1	2.8	5.6	4.8	2.1	2.8
		x		13.0	11.4	24.6	24.3	8.3	-4.9	2.5	1.3	8.3	4.9	2.5	1.3
			x	13.2	12.0	24.6	24.4	10.0	0.2	2.5	1.7	10.0	0.2	2.5	1.7
x	x			11.1	13.9	25.2	25.7	-7.3	15.5	4.9	7.0	7.3	15.5	4.9	7.0
x		x		12.7	13.8	25.3	25.3	5.7	15.0	5.4	5.5	5.7	15.0	5.4	5.5
x			x	13.0	14.4	25.3	25.4	8.6	20.0	5.3	5.8	8.6	20.0	5.3	5.8
	x	x		12.4	10.8	25.1	25.0	3.1	-10.0	4.7	4.2	3.1	10.0	4.7	4.2
	x		x	12.5	11.4	25.1	25.1	4.4	-5.0	4.6	4.5	4.4	5.0	4.6	4.5
		x	x	14.2	11.5	25.2	24.7	18.3	-4.4	5.1	3.0	18.3	4.4	5.1	3.0
	x	x	x	13.6	10.8	25.7	25.4	13.0	-10.0	7.2	5.8	13.0	10.0	7.2	5.8
x		x	x	13.9	13.8	25.9	25.7	15.7	15.3	7.9	7.2	15.7	15.3	7.9	7.2
x	x	x		12.0	13.2	25.8	26.0	0.0	10.2	7.5	8.3	0.0	10.2	7.5	8.3
x	x		x	12.3	13.8	25.8	26.1	2.6	15.0	7.5	8.6	2.6	15.0	7.5	8.6
x	x	x	x	13.2	13.2	26.4	26.4	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
			Average	12.7	12.7	25.3	25.3	5.7	5.5	5.3	5.3	7.6	10.7	5.3	5.3
			Max	14.2	14.4	26.4	26.4	18.3	20.0	10.0	10.0	18.3	20.0	10.0	10.0
			Min	11.1	10.8	24.5	24.3	-7.3	-10.0	2.1	1.3	0.0	0.2	2.1	1.3

*Out of phase π/4

Serie 3 4

0	$2\pi/3$	$4\pi/3$	$\pi/2$	r *	r	t *	t	% r*	% r	% t *	% t	|% r*|	|% r|	|%t *|	|%t|
x				11.3	14.0	24.5	24.5	-5.6	16.7	1.9	1.9	5.6	16.7	1.9	1.9
	x			12.5	11.8	24.7	24.7	3.7	-1.3	2.8	2.8	3.7	1.3	2.8	2.8
		x		12.5	11.8	24.7	24.7	3.7	-1.3	2.8	2.8	3.7	1.3	2.8	2.8
			x	13.2	11.6	24.6	24.6	10.0	-3.3	2.5	2.5	10.0	3.3	2.5	2.5
x	x			11.8	13.8	25.1	25.1	-1.8	15.3	4.7	4.7	1.8	15.3	4.7	4.7
x		x		11.8	13.8	25.1	25.1	-1.8	15.3	4.7	4.7	1.8	15.3	4.7	4.7
x			x	12.5	13.6	25.1	25.1	4.4	13.3	4.4	4.4	4.4	13.3	4.4	4.4
	x	x		12.7	11.6	25.3	25.3	5.5	-3.3	5.5	5.5	5.5	3.3	5.5	5.5
	x		x	13.6	11.4	25.3	25.3	13.7	-4.7	5.3	5.3	13.7	4.7	5.3	5.3
		x	x	13.6	11.4	25.3	25.3	13.7	-4.7	5.3	5.3	13.7	4.7	5.3	5.3
	x	x	x	13.9	11.2	25.9	25.9	15.5	-6.7	8.0	8.0	15.5	6.7	8.0	8.0
x		x	x	13.0	13.4	25.7	25.7	8.2	12.0	7.2	7.2	8.2	12.0	7.2	7.2
x	x	x		12.0	13.6	25.8	25.8	0.0	13.3	7.5	7.5	0.0	13.3	7.5	7.5
x	x		x	13.0	13.4	25.7	25.7	8.2	12.0	7.2	7.2	8.2	12.0	7.2	7.2
x	x	x	x	13.2	13.2	26.4	26.4	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
			Average	12.7	12.7	25.3	25.3	5.8	5.5	5.3	5.3	7.0	8.9	5.3	5.3
			Max	13.9	14.0	26.4	26.4	15.5	16.7	10.0	10.0	15.5	16.7	10.0	10.0
			Min	11.3	11.2	24.5	24.5	-5.6	-6.7	1.9	1.9	0.0	1.3	1.9	1.9

*Out of phase $\pi/2$

Serie 5 6

0	π/4	π/2	3π/4	r *	r	t *	t	% r*	% r	% t *	% t	|% r*|	|% r|	|%t *|	|%t|
x				13.2	13.8	24.8	24.9	10.0	15.0	3.3	3.7	10.0	15.0	3.3	3.7
	x			11.5	12.0	24.4	24.6	-4.2	0.0	1.7	2.5	4.2	0.0	1.7	2.5
		x		11.5	11.4	24.4	24.3	-4.2	-5.0	1.7	1.3	4.2	5.0	1.7	1.3
			x	13.2	12.0	24.8	24.6	10.0	0.0	3.3	2.5	10.0	0.0	3.3	2.5
x	x			12.7	13.8	25.2	25.5	5.8	15.0	5.0	6.3	5.8	15.0	5.0	6.3
x		x		12.6	13.2	25.2	25.2	5.0	10.0	5.0	5.0	5.0	10.0	5.0	5.0
x			x	14.3	13.8	25.6	25.5	19.2	15.0	6.7	6.3	19.2	15.0	6.7	6.3
	x	x		10.9	11.5	24.8	24.9	-9.2	-4.2	3.3	3.7	9.2	4.2	3.3	3.7
	x		x	12.6	12.0	25.2	25.2	5.0	0.0	5.0	5.0	5.0	0.0	5.0	5.0
		x	x	12.7	11.5	25.2	24.9	5.8	-4.2	5.0	3.7	5.8	4.2	5.0	3.7
	x	x	x	12.1	11.4	25.6	25.5	0.8	-5.0	6.7	6.3	0.8	5.0	6.7	6.3
x		x	x	13.8	13.2	26.0	25.8	15.0	10.0	8.3	7.5	15.0	10.0	8.3	7.5
x	x	x		12.1	13.2	25.6	25.8	0.8	10.0	6.7	7.5	0.8	10.0	6.7	7.5
x	x		x	13.0	13.8	26.0	26.1	8.3	15.0	8.3	8.8	8.3	15.0	8.3	8.8
x	x	x	x	13.2	13.2	26.4	26.4	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
			Average	12.6	12.7	25.3	25.3	5.2	5.4	5.3	5.3	7.6	7.9	5.3	5.3
			Max	14.3	13.8	26.4	26.4	19.2	15.0	10.0	10.0	19.2	15.0	10.0	10.0
			Min	10.9	11.4	24.4	24.3	-9.2	-5.0	1.7	1.3	0.8	0.0	1.7	1.3

*Out of phase π/8

Serie 7 8

0	π/4	π/2	3π/4	r π/4*	r π/3*	t π/4	t π/3	% r π/4	% r π/3	% t π/4	% t π/3	|%r| π/4	|% r| π/3	|%t| π/4	|% t| π/3
x				12	11.6	24.6	24.5	0.0	-3.3	2.5	2.1	0.0	3.3	2.5	2.1
	x			11.4	11.4	24.3	24.3	-5.0	-5.0	1.3	1.3	5.0	5.0	1.3	1.3
		x		12	12.8	24.6	24.7	0.0	6.7	2.5	2.9	0.0	6.7	2.5	2.9
			x	13.8	13.5	24.9	24.8	15.0	12.5	3.7	3.3	15.0	12.5	3.7	3.3
x	x			11.5	11	24.9	24.8	-4.2	-8.3	3.7	3.3	4.2	8.3	3.7	3.3
x		x		12	12.3	25.2	25.2	0.0	2.5	5.0	5.0	0.0	2.5	5.0	5.0
x			x	13.8	12.2	25.5	25.3	15.0	1.7	6.3	5.4	15.0	1.7	6.3	5.4
	x	x		11.5	12.3	24.9	25.1	-4.2	2.5	3.7	4.6	4.2	2.5	3.7	4.6
	x		x	13.2	12.9	25.2	25.2	10.0	7.5	5.0	5.0	10.0	7.5	5.0	5.0
		x	x	13.8	14.2	25.5	25.6	15.0	18.3	6.3	6.7	15.0	18.3	6.3	6.7
	x	x	x	13.2	11.8	25.8	25.6	10.0	-1.7	7.5	6.7	10.0	1.7	7.5	6.7
x		x	x	13.8	12.5	26.1	25.6	15.0	4.2	8.8	6.7	15.0	4.2	8.8	6.7
x	x	x		11.4	11.7	25.5	25.5	-5.0	-2.5	6.3	6.3	5.0	2.5	6.3	6.3
x	x		x	13.2	12.5	25.8	25.6	10.0	4.2	7.5	6.7	10.0	4.2	7.5	6.7
x	x	x	x	13.2	13.2	26.4	26.4	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
			Average	12.7	12.4	25.3	25.2	5.4	3.3	5.3	5.1	7.9	6.1	5.3	5.1
			Max	13.8	14.2	26.4	26.4	15.0	18.3	10.0	10.0	15.0	18.3	10.0	10.0
			Min	11.4	11.0	24.3	24.3	-5.0	-8.3	1.3	1.3	0.0	1.7	1.3	1.3

*Out of phase

APPENDIX C

CODES

I MAIN PROGRAM

Program characterisation

```
        use  Msflib
        USE DIALOGM

        real      a1,a2,a3,t1,t2,t3,t0,t4,t5,r0,r1,r2,r3,r4,r5,rc0,rs0,rc4,rs4,b1,b2,b3,b4
        ,b5,b6,t10,t14,rc10,rs10,rc14,rs14,c1,c2,c3,c4,c5,c6

        include 'resource.fd'
        TYPE (DIALOG)  dlg
        Logical return

        return = DLGINIT(IDD_DIALOG1,dlg)

        retint =DlgModal(dlg)

        CALL DlgUninit(dlg)

1       a1=0

        call combo(t1,t2,t3,r1,r2,r3,t5,r5,a2,a3)
        Call calcul(t1,t2,t3,t5,r5,r1,r2,r3,a1,a2,a3,t0,t4,rc0,rs0,rc4,rs4,r0,r4)
        Call cartesian (t0,t4,rc0,rs0,rc4,rs4,b1,b2,b3,b4,b5,b6)
        Call inverse(t0,t4,rc0,rs0,rc4,rs4,r0,r4,t10,t14,rc10,rs10,rc14,rs14)
        Call cartesian1(t10,t14,rc10,rs10,rc14,rs14,c1,c2,c3,c4,c5,c6)
end
```

II WAY TO ENTER THE VALUES

```
subroutine combo (t1,t2,t3,r1,r2,r3,t5,r5,a2,a3)

use msflib
use dialogm
implicit none
include 'resource.fd'
Logical retlog
Logical return
Logical retint
```

```
character  n*12
CHARACTER(256) str
integer num,d,c
real t1,t2,t3,r1,r2,r3,t5,r5,a2,a3
type(dialog) dlg
return=DLGINIT (IDD_DIALOG3,dlg)
retlog = DLGSET (dlg, IDC_COMBO1,3,dlg_numitems)
retlog = DLGSET (dlg, IDC_COMBO1,"Read the values in a file",1)
retlog = DLGSET (dlg, IDC_COMBO1, "Create a file",2)
retlog = DLGSET (dlg, IDC_COMBO1, "Enter the values",3)
retlog =DLGGET (dlg,IDC_COMBO1,str,DLG_STATE)

 retint =dlgmodal (dlg)
 retlog =DLGGET (dlg,IDC_COMBO1,num,DLG_STATE)
 retlog =DLGGET (dlg,IDC_COMBO1,str,DLG_STATE)
 Call DlgUninit(Dlg)
 c=num
```

II.1 Read the values in a file

```
        if (c.eq.3) then
                call edit7 (n)
                Call  values (t1,t2,t3,r1,r2,r3,t5,r5,a2,a3,n)
                goto 1
        end if
```

II.2 Create a file

```
        if (c.eq.1) then
                call clearscreen ($GCLEARSCREEN)
                call combo2(d)

                if (d.eq.1) then
                        Call   creation1(n)
                        Call  values (t1,t2,t3,r1,r2,r3,t5,r5,a2,a3,n)
                        goto 1
                end if

                if (d.eq.3)       then
                        Call   creation2(n)
                        Call  values (t1,t2,t3,r1,r2,r3,t5,r5,a2,a3,n)
                        goto 1
                end if

                if (d.eq.2)       then
                        Call   creation3(n)
                        Call  values (t1,t2,t3,r1,r2,r3,t5,r5,a2,a3,n)
                        goto 1
                end if
        end if
```

II.3 Enter directly the values

```
        if (c.eq.2) then
                call clearscreen ($GCLEARSCREEN)
                call combo2(d)

                if (d.eq.1) then
                        Call    results  (t1,t2,t3,r1,r2,r3,t5,r5,a2,a3)
                        goto 1
                end if

                if (d.eq.3) then
                        Call  results1(t1,t2,t3,r1,r2,r3,t5,r5,a2,a3)
                        goto 1
                end if

                if (d.eq.2) then
                        Call  results2 (t1,t2,t3,r1,r2,r3,t5,r5,a2,a3)
                        goto 1
                end if

        end if

1 End subroutine combo
```

III NAME OF THE FILE

```
subroutine edit7 (n)

        use msflib
        use dialogm
        implicit none
        include 'resource.fd'
        Logical retlog
        Logical return
        Logical retint
        Character(256) text1
        Character(256) n

        type(dialog) dlg
        return=DLGINIT (IDD_DIALOG11,dlg)

1       retint =dlgmodal (dlg)
        retlog =DLGGET (dlg,IDC_EDIT1,text1)
        n= text1

        if (n.eq." ") goto 1
 end
```

IV FORM OF THE RESULTS

```
subroutine combo2(d)

use msflib
use dialogm
implicit none
include 'resource.fd'
Logical retlog
Logical return
Logical retint

CHARACTER(256) str
integer num,d

type(dialog) dlg
return=DLGINIT (IDD_DIALOG4,dlg)

retlog = DLGSET (dlg, IDC_COMBO2,3,dlg_numitems)
retlog = DLGSET (dlg, IDC_COMBO2,"You know the polar coordinates of your
results",1)
retlog = DLGSET (dlg, IDC_COMBO2, "You use two gages for each sample",2)
retlog = DLGSET (dlg, IDC_COMBO2, "You use four gages for each sample",3)
retlog =DLGGET (dlg,IDC_COMBO2,str,DLG_STATE)

 retint =dlgmodal (dlg)
 retlog =DLGGET (dlg,IDC_COMBO2,num,DLG_STATE)
 retlog =DLGGET (dlg,IDC_COMBO2,str,DLG_STATE)
 Call DlgUninit(Dlg)

 d=num

 end
```

V ENTER THE VALUES

V.1 Stress applied and angles between the samples

```
subroutine edit1(t,a2,a3)

use msflib
use dialogm
implicit none
include 'resource.fd'
Logical retlog
Logical return
Logical retint
```

```
Character(256) text1
Character(256) text2
Character(256) text3
real t,a2,a3

type(dialog) dlg
 return=DLGINIT (IDD_DIALOG5,dlg)

1 retint =dlgmodal (dlg)
 retlog =DLGGET (dlg,IDC_EDIT1,text1)
 read(text1,*) t
 retlog =DLGGET (dlg,IDC_edit2,text2)
 read(text2,*) a2

 retlog =DLGGET (dlg,IDC_edit3,text3)
 read(text3,*) a3
 if (t.eq.0) goto 1
 if (a2.eq.0) goto 1
 if (a3.eq.0) goto 1

 Call DlgUninit(Dlg)

 end
```

V.2 Form of the results : Polar

```
subroutine edit2(t1,t2,t3,r1,r2,r3)

        use msflib
        use dialogm
        implicit none
        include 'resource.fd'
        Logical retlog
        Logical return
        Logical retint
        Character(256) text1
        Character(256) text2
        Character(256) text3
        Character(256) text4
        Character(256) text5
        Character(256) text6

        real t1,t2,t3,r1,r2,r3

        type(dialog) dlg
        return=DLGINIT (IDD_DIALOG6,dlg)
```

```
1       retint =dlgmodal (dlg)
        retlog =DLGGET (dlg,IDC_EDIT2,text1)
        read(text1,*) t1
        retlog =DLGGET (dlg,IDC_edit3,text2)
        read(text2,*) r1

        retlog =DLGGET (dlg,IDC_edit4,text3)
        read(text3,*) t2

        retlog =DLGGET (dlg,IDC_edit5,text4)
        read(text4,*) r2

        retlog =DLGGET (dlg,IDC_edit6,text5)
        read(text5,*) t3

        retlog =DLGGET (dlg,IDC_edit7,text6)
        read(text6,*) r3
        if (t1.eq.0) goto 1
        if (t2.eq.0) goto 1
        if (t3.eq.0) goto 1
        if (r1.eq.0) goto 1
        if (r2.eq.0) goto 1
        if (r3.eq.0) goto 1

 Call DlgUninit(Dlg)

 end
```

V.3 Form of the results : Two gages

```
        subroutine edit3 ( x1,y1,x2,y2,x3,y3 )

        use msflib
        use dialogm
        implicit none
        include 'resource.fd'
        Logical retlog
        Logical return
        Logical retint
        Character(256) text1
        Character(256) text2
        Character(256) text3
        Character(256) text4
        Character(256) text5
        Character(256) text6

        real x1,x2,x3,y1,y2,y3
```

```
      type(dialog) dlg
      return=DLGINIT (IDD_DIALOG7,dlg)

1     retint =dlgmodal (dlg)
      retlog =DLGGET (dlg,IDC_EDIT1,text1)
      read(text1,*) x1
      retlog =DLGGET (dlg,IDC_edit2,text2)
      read(text2,*) y1

      retlog =DLGGET (dlg,IDC_edit3,text3)
      read(text3,*) x2

      retlog =DLGGET (dlg,IDC_edit4,text4)
      read(text4,*) y2

      retlog =DLGGET (dlg,IDC_edit5,text5)
      read(text5,*) x3

      retlog =DLGGET (dlg,IDC_edit6,text6)
      read(text6,*) y3
      if (x1.eq.0) goto 1
      if (x2.eq.0) goto 1
      if (x3.eq.0) goto 1
      if (y1.eq.0) goto 1
      if (y2.eq.0) goto 1
      if (y3.eq.0) goto 1

      Call DlgUninit(Dlg)

end
```

V.4 Form of the results : Four gages

V.4.1 First sample

```
      subroutine edit4 ( w1,x1,y1,z1)

      use msflib
      use dialogm
      implicit none
      include 'resource.fd'
      Logical retlog
      Logical return
      Logical retint
      Character(256) text1
      Character(256) text2
      Character(256) text3
```

```
        Character(256) text4
        real w1,x1,y1,z1

        type(dialog) dlg
        return=DLGINIT (IDD_DIALOG8,dlg)

1       retint =dlgmodal (dlg)
        retlog =DLGGET (dlg,IDC_EDIT1,text1)
        read(text1,*) w1
        retlog =DLGGET (dlg,IDC_edit2,text2)
        read(text2,*) x1

        retlog =DLGGET (dlg,IDC_edit3,text3)
        read(text3,*) y1

        retlog =DLGGET (dlg,IDC_edit4,text4)
        read(text4,*) z1
        if (w1.eq.0) goto 1
        if (x1.eq.0) goto 1
        if (y1.eq.0) goto 1
        if (z1.eq.0) goto 1

        Call DlgUninit(Dlg)

 end
```

V.4.2 Second sample

```
subroutine edit5( w1,x1,y1,z1)

        use msflib
        use dialogm
        implicit none
        include 'resource.fd'
        Logical retlog
        Logical return
        Logical retint
        Character(256) text1
        Character(256) text2
        Character(256) text3
        Character(256) text4

        real w1,x1,y1,z1

        type(dialog) dlg
        return=DLGINIT (IDD_DIALOG9,dlg)
```

```
1       retint =dlgmodal (dlg)
        retlog =DLGGET (dlg,IDC_EDIT1,text1)
         read(text1,*) w1
         retlog =DLGGET (dlg,IDC_edit2,text2)
         read(text2,*) x1

         retlog =DLGGET (dlg,IDC_edit3,text3)
        read(text3,*) y1

        retlog =DLGGET (dlg,IDC_edit4,text4)
        read(text4,*) z1
        if (w1.eq.0) goto 1
        if (x1.eq.0) goto 1
        if (y1.eq.0) goto 1
        if (z1.eq.0) goto 1

        Call DlgUninit(Dlg)

 end
```

V.4.3 Third sample

```
subroutine edit6( w1,x1,y1,z1)

        use msflib
        use dialogm
        implicit none
        include 'resource.fd'
        Logical retlog
        Logical return
        Logical retint
        Character(256) text1
        Character(256) text2
        Character(256) text3
        Character(256) text4

        real w1,x1,y1,z1

        type(dialog) dlg
        return=DLGINIT (IDD_DIALOG10,dlg)

1       retint =dlgmodal (dlg)
        retlog =DLGGET (dlg,IDC_EDIT1,text1)
        read(text1,*) w1
        retlog =DLGGET (dlg,IDC_edit2,text2)
        read(text2,*) x1
```

```
        retlog =DLGGET (dlg,IDC_edit3,text3)
        read(text3,*) y1

        retlog =DLGGET (dlg,IDC_edit4,text4)
        read(text4,*) z1
        if (w1.eq.0) goto 1
        if (x1.eq.0) goto 1
        if (y1.eq.0) goto 1
        if (z1.eq.0) goto 1

        Call DlgUninit(Dlg)

end
```

VI POLAR COMPONENTS CALCULATION

VI.1 Form of the results : Polar

```
subroutine results (t1,t2,t3,r1,r2,r3,t5,r5,a2,a3)

        real t

        call edit1(t,a2,a3)

        t5=t/2
        r5=t/2

        Call edit2(t1,t2,t3,r1,r2,r3)

         write (*,*) t1,t2,t3,r1,r2,r3,t5,r5,a2,a3
Endsubroutine results
```

VI.2 Form of the results : Two gages

```
subroutine results1 (t1,t2,t3,r1,r2,r3,t5,r5,a2,a3)

        real t,x1,y1,x2,y2,x3,y3

        call edit1(t,a2,a3)
        t5=t/2
        r5=t/2

        call edit3 ( x1,y1,x2,y2,x3,y3 )
        t1=(x1+y1)/2
        r1=(x1-y1)/2
```

```
        t2=(x2+y2)/2
        r2=(x2-y2)/2


        t3=(x3+y3)/2
        r3=(x3-y3)/2
        WRITE(*,*)T1,R1,T2,R2,T3,R3
Endsubroutine results1
```

VI.3 Form of the results : Four gages

```
subroutine results2 (t1,t2,t3,r1,r2,r3,t5,r5,a2,a3)

         real t,w1,z1,x1,y1,w2,z2,x2,y2,w3,z3,x3,y3

        call edit1(t,a2,a3)
        t5=t/2
        r5=t/2

        call edit4 ( w1,x1,y1,z1)
        t1=0.25*w1+0.25*x1+0.25*y1+0.25*z1
        r1=0.5*w1-0.5*y1

        call edit5( w2,x2,y2,z2)
        t2=0.25*w2+0.25*x2+0.25*y2+0.25*z2
        r2=0.5*w2-0.5*y2

        call edit6( w3,x3,y3,z3)
        t3=0.25*w3+0.25*x3+0.25*y3+0.25*z3
        r3=0.5*w3-0.5*y3

Endsubroutine results2
```

VII VALUES IN FILE

VII.1 Form of the results : Polar

```
subroutine creation1(n)
        real t
        character  n*12

        call edit7 (n)

        call edit1(t,a2,a3)
```

```
t5=t/2
r5=t/2

call edit2(t1,t2,t3,r1,r2,r3)

open(1,FILE=n,STATUS="replace",
     ACCESS="direct",FORM="unformatted",recl=50)

WRITE(1)t1,t2,t3,r1,r2,r3,t5,r5,a2,a3

close(1)

End
```

VII.2 Form of the results : Two gages

```
subroutine creation2(n)

    character  n*12
    real t,x1,y1,x2,y2,x3,y3

    call edit7 (n)

    call edit1(t,a2,a3)

    t5=t/2
    r5=t/2
    call edit3 ( x1,y1,x2,y2,x3,y3 )

    t1=(x1+y1)/2
    r1=(x1-y1)/2

    t2=(x2+y2)/2
    r2=(x2-y2)/2

    t3=(x3+y3)/2
    r3=(x3-y3)/2

    open(1,FILE=n,STATUS="replace",ACCESS="direct",FORM="unformatted"
,recl=50)

    WRITE(1)t1,t2,t3,r1,r2,r3,t5,r5,a2,a3

    close(1)

    End
```

VII.3 Form of the results : Four gages

```
subroutine creation3(n)

        character  n*12
        real t,w1,z1,x1,y1,w2,z2,x2,y2,w3,z3,x3,y3

        call edit7 (n)

        call edit1(t,a2,a3)

        t5=t/2
        r5=t/2

        call edit4 ( w1,x1,y1,z1)

        t1=0.25*w1+0.25*x1+0.25*y1+0.25*z1
        r1=0.5*w1-0.5*y1

        call edit5( w2,x2,y2,z2)
        t2=0.25*w2+0.25*x2+0.25*y2+0.25*z2
        r2=0.5*w2-0.5*y2

        call edit6( w3,x3,y3,z3)

        t3=0.25*w3+0.25*x3+0.25*y3+0.25*z3
        r3=0.5*w3-0.5*y3

        open(1,FILE=n,STATUS="replace",ACCESS="direct",FORM="unformatted"
,recl=50)

        WRITE(1)t1,t2,t3,r1,r2,r3,t5,r5,a2,a3

        close(1)

End
```

VIII READ VALUES IN FILE

```
subroutine values (t1,t2,t3,r1,r2,r3,t5,r5,a2,a3,n)

character  n*12

open(1,FILE=n,STATUS="old",ACCESS="direct",FORM="unformatted",recl=50)

read(1)t1,t2,t3,r1,r2,r3,t5,r5,a2,a3
Close(1)
```

```
write (*,*)      t1,t2,t3,r1,r2,r3,t5,r5,a2,a3

End
```

IX SOLVE SYSTEM

```
subroutine calcul (t1,t2,t3,t5,r5,r1,r2,r3,a1,a2,a3,t0,t4,rc0,rs0,rc4,rs4,r0,r4)

  real c1,c2,c3,s1,s2,s3,c4,c5,c6,s4,s5,s6
  c1 = COS(a2-a3)
  c2 = COS(a1-a3)
  c3 = COS(a1-a2)
  c4 = COS(a2+a3)
  c5 = COS(a1+a3)
  c6 = COS(a1+a2)
  s1 = SIN(a2-a3)
  s2 = SIN(a1-a3)
  s3 = SIN(a1-a2)
  s4 = SIN(a2+a3)
  s5 = SIN(a1+a3)
  s6 = SIN(a1+a2)


  t4=c1/(8*t5*s3*s2)*t1-c2/(8*t5*s3*s1)*t2+c3/(8*t5*s2*s1)*t3


        rc4=-c4/(8*r5*s3*s2)*t1+c5/(8*r5*s3*s1)*t2-c6/(8*r5*s2*s1)*t3


        rs4=s4/(8*r5*s3*s2)*t1-s5/(8*r5*s3*s1)*t2+s6/(8*r5*s2*s1)*t3


        r4=sqrt(rc4*rc4+rs4*rs4)



        t0=t5*(c3*c3-s3*s3+c2*c2-s2*s2+1)/(16*r5*r5*c2*c3*s3*s2)*t1
        t0=t0-t5*(c3*c3-s3*s3+c1*c1-s1*s1+1)/(16*r5*r5*c1*c3*s3*s1)*t2
        t0=t0+t5*(c2*c2-s2*s2+c1*c1-s1*s1+1)/(16*r5*r5*c1*c2*s2*s1)*t3
        t0=t0+(c1*c1-s1*s1)/(4*r5*(2*c3*s3)*(2*c2*s2))*r1
        t0=t0-(c2*c2-s2*s2)/(4*r5*(2*c3*s3)*(2*c1*s1))*r2
        t0=t0+(c3*c3-s3*s3)/(4*r5*(2*c2*s2)*(2*c1*s1))*r3

   rc0=t5*(c4*c4-s4*s4)/(4*r5*r5*(2*c2*s2)*(2*c3*s3))*t1
        rc0=rc0-t5*(c5*c5-s5*s5)/(4*r5*r5*(2*c1*s1)*(2*c3*s3))*t2
        rc0=rc0+ t5*(c6*c6-s6*s6)/(4*r5*r5*(2*c1*s1)*(2*c2*s2))*t3
        rc0=rc0-(c4*c4-s4*s4)/(4*r5*(2*c3*s3)*(2*c2*s2))*r1
```

```
        rc0=rc0+(c5*c5-s5*s5)/(4*r5*(2*c3*s3)*(2*c1*s1))*r2
        rc0=rc0- (c6*c6-s6*s6)/(4*r5*(2*c2*s2)*(2*c1*s1))*r3


        rs0=-t5*(2*s4*c4)/(4*r5*r5*(2*c2*s2)*(2*c3*s3))*t1
        rs0=rs0+t5*(2*c5*s5)/(4*r5*r5*(2*c1*s1)*(2*c3*s3))*t2
        rs0=rs0- t5*(2*c6*s6)/(4*r5*r5*(2*c1*s1)*(2*c2*s2))*t3
        rs0=rs0+(2*c4*s4)/(4*r5*(2*c3*s3)*(2*c2*s2))*r1
        rs0=rs0-(2*c5*s5)/(4*r5*(2*c3*s3)*(2*c1*s1))*r2
        rs0=rs0+ (2*c6*s6)/(4*r5*(2*c2*s2)*(2*c1*s1))*r3


        r0=sqrt(rc0*rc0+rs0*rs0)

        write(*,*) "For compliance "

        write(*,*) "t1=",t4
        write(*,*) "r1 cos(2a1)=",rc4
        write(*,*) "r1 sin(2a1)=",rs4
        write(*,*) "r1=",r4
        write(*,*) "t0=",t0
        write(*,*) "r0 cos(4a0)=",rc0
        write(*,*) "r0 sin(4 a0)=",rs0
        write(*,*) "r0=",r0

  call edit8(t0,rs0,rc0,t4,rs4,rc4)
  End
```

X EDIT POLAR COMPONENTS OF THE COMPLIANCE MATRIX

```
subroutine edit8 (t0,rs0,rc0,t4,rs4,rc4)

 use msflib
use dialogm
implicit none
include 'resource.fd'
Logical retlog
Logical return
Logical retint

Character(256) text1
Character(256) text2
Character(256) text3
Character(256) text4
Character(256) text5
```

```
Character(256) text6

real t0,rs0,rc0,t4,rs4,rc4

type(dialog) dlg
return=DLGINIT (IDD_DIALOG12,dlg)

 write(text1,*) t4
 retlog =DLGSET (dlg,IDC_EDIT1,trim(adjustl(text1)))

 write(text2,*) rc4
 retlog =DLGSET (dlg,IDC_edit2,trim(adjustl(text2)))

 write(text3,*) rs4
 retlog =DLGSET (dlg,IDC_edit3,trim(adjustl(text3)))

 write(text4,*) t0
 retlog =DLGSET (dlg,IDC_edit4,trim(adjustl(text4)))

 write(text5,*) rc0
 retlog =DLGSET (dlg,IDC_edit5,trim(adjustl(text5)))

  write(text6,*) rs0
  retlog =DLGSET (dlg,IDC_edit6,trim(adjustl(text6)))
  retint =dlgmodal (dlg)

 Call DlgUninit(Dlg)

 end
```

XI CARTESIAN COORDINATES OF THE COMPLIANCE MATRIX

XI.1 Calculate

```
subroutine cartesian(t0,t4,rc0,rs0,rc4,rs4,b1,b2,b3,b4,b5,b6)

          b1=     t0+2*t4+rc0+4*rc4
          b2=-t0+2*t4-rc0
          b3=t0+2*t4+rc0-4*rc4
          b4=(t0-rc0)*4
          b5=(rs0+2*rs4)*2
          b6=(-rs0+2*rs4)*2

          call edit10( b1,b2,b3,b4,b5,b6)
```

```
        Write (*,*)"a11=",b1
        Write (*,*)"a12=",b2
        Write (*,*)"a22=",b3
        Write (*,*)"a66=",b4
        Write (*,*)"a16=",b5
        Write (*,*)"a62=",b6

    end
```

XI.2 Edit

```
subroutine edit10( b1,b2,b3,b4,b5,b6)

 use msflib
use dialogm
implicit none
include 'resource.fd'
Logical retlog
Logical return
Logical retint

Character(256) text1
Character(256) text2
Character(256) text3
Character(256) text4
Character(256) text5
Character(256) text6
Character(256) text7
Character(256) text8
Character(256) text9

real b1,b2,b3,b4,b5,b6

type(dialog) dlg
return=DLGINIT (IDD_DIALOG14,dlg)

 write(text1,*) b1
 retlog =DLGSET (dlg,IDC_EDIT1,trim(adjustl(text1)))

 write(text2,*) b2
 retlog =DLGSET (dlg,IDC_edit2,trim(adjustl(text2)))

 write(text3,*) b5
 retlog =DLGSET (dlg,IDC_edit3,trim(adjustl(text3)))

 write(text4,*) b2
 retlog =DLGSET (dlg,IDC_edit4,trim(adjustl(text4)))
```

```
write(text5,*) b3
retlog =DLGSET (dlg,IDC_edit5,trim(adjustl(text5)))

 write(text6,*) b6
 retlog =DLGSET (dlg,IDC_edit6,trim(adjustl(text6)))

 write(text7,*) b5
retlog =DLGSET (dlg,IDC_edit7,trim(adjustl(text7)))

write(text8,*) b6
retlog =DLGSET (dlg,IDC_edit8,trim(adjustl(text8)))

 write(text9,*) b4
 retlog =DLGSET (dlg,IDC_edit9,trim(adjustl(text9)))

 retint =dlgmodal (dlg)

Call DlgUninit(Dlg)

end
```

XII POLAR COMPONENTS OF THE STIFFNESS MATRIX

XII.1 Calculate

```
subroutine inverse(t0,t4,rc0,rs0,rc4,rs4,r0,r4,t10,t14,rc10,rs10,rc14,rs14)

real q
q=16*t0*t0*t4-16*t4*r0*r0-32*t0*r4*r4+32*(rc0*(rc4*rc4-rs4*rs4)+2*rs0*rc4*rs4)

t14=(t0*t0-r0*r0)/q
t10=4*(t0*t4-r4*r4)/q
rc14=(2*(rc4*rc0+rs4*rs0)-2*t0*rc4)/q
rs14=(2*(-rc0*rs4+rs0*rc4)-2*t0*rs4)/q
rc10=(-4*t4*rc0+4*(rc4*rc4-rs4*rs4))/q
rs10=(-4*t4*rs0+4*(2*rc4*rs4))/q

call edit9(t10,t14,rc10,rs10,rc14,rs14)
        write(*,*) "For stiffness "
        write(*,*) "t1=",t14
        write(*,*) "r1 cos(2a1)=",rc14
        write(*,*) "r1 sin(2a1)=",rs14
        write(*,*) "t0=",t10
        write(*,*) "r0 cos(4a0)=",rc10
        write(*,*) "r0 sin(4 a0)=",rs10
end
```

XII.2 Edit

```
subroutine edit9 ( t10,t14,rc10,rs10,rc14,rs14)

 use msflib
use dialogm
implicit none
include 'resource.fd'
Logical retlog
Logical return
Logical retint

Character(256) text1
Character(256) text2
Character(256) text3
Character(256) text4
Character(256) text5
Character(256) text6

real t10,rs10,rc10,t14,rs14,rc14

type(dialog) dlg
return=DLGINIT (IDD_DIALOG13,dlg)

 write(text1,*) t14
 retlog =DLGSET (dlg,IDC_EDIT1,trim(adjustl(text1)))

 write(text2,*) rc14
 retlog =DLGSET (dlg,IDC_edit2,trim(adjustl(text2)))

 write(text3,*) rs14
 retlog =DLGSET (dlg,IDC_edit3,trim(adjustl(text3)))

 write(text4,*) t10
 retlog =DLGSET (dlg,IDC_edit4,trim(adjustl(text4)))

 write(text5,*) rc10
 retlog =DLGSET (dlg,IDC_edit5,trim(adjustl(text5)))

  write(text6,*) rs10
  retlog =DLGSET (dlg,IDC_edit6,trim(adjustl(text6)))

  retint =dlgmodal (dlg)

 Call DlgUninit(Dlg)

 end
```

XIII CARTESIAN COORDINATES OF THE COMPLIANCE MATRIX

XIII.1 Calculate

```
subroutine cartesian1(t0,t4,rc0,rs0,rc4,rs4,b1,b2,b3,b4,b5,b6)
                    b1=     t0+2*t4+rc0+4*rc4
                    b2=-t0+2*t4-rc0
                    b3=t0+2*t4+rc0-4*rc4
                    b4=(t0-rc0)
                    b5=(rs0+2*rs4)
                    b6=(-rs0+2*rs4)

                    call edit11( b1,b2,b3,b4,b5,b6)

                    Write (*,*)"A11=",b1
                    Write (*,*)"A12=",b2
                    Write (*,*)"A22=",b3
                    Write (*,*)"A66=",b4
                    Write (*,*)"A16=",b5
                    Write (*,*)"A62=",b6

          end
```

XIII.2 Edit

```
subroutine edit11( b1,b2,b3,b4,b5,b6)

 use msflib
use dialogm
implicit none
include 'resource.fd'
Logical retlog
Logical return
Logical retint

Character(256) text1
Character(256) text2
Character(256) text3
Character(256) text4
Character(256) text5
Character(256) text6
Character(256) text7
Character(256) text8
Character(256) text9
```

```
real b1,b2,b3,b4,b5,b6

type(dialog) dlg
return=DLGINIT (IDD_DIALOG15,dlg)

 write(text1,*) b1
 retlog =DLGSET (dlg,IDC_EDIT1,trim(adjustl(text1)))

 write(text2,*) b2
 retlog =DLGSET (dlg,IDC_edit2,trim(adjustl(text2)))

 write(text3,*) b5
 retlog =DLGSET (dlg,IDC_edit3,trim(adjustl(text3)))

 write(text4,*) b2
 retlog =DLGSET (dlg,IDC_edit4,trim(adjustl(text4)))

 write(text5,*) b3
 retlog =DLGSET (dlg,IDC_edit5,trim(adjustl(text5)))

  write(text6,*) b6
  retlog =DLGSET (dlg,IDC_edit6,trim(adjustl(text6)))

  write(text7,*) b5
 retlog =DLGSET (dlg,IDC_edit7,trim(adjustl(text7)))

 write(text8,*) b6
 retlog =DLGSET (dlg,IDC_edit8,trim(adjustl(text8)))

  write(text9,*) b4
  retlog =DLGSET (dlg,IDC_edit9,trim(adjustl(text9)))

  retint =dlgmodal (dlg)

 Call DlgUninit(Dlg)

 end
```

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